PHD

An investigation into the use of constraint modelling techniques in the design of packaging machinery motion

Twyman, Benjamin Roger

Award date:
1999

Awarding institution:
University of Bath

An investigation into the use of constraint modelling techniques in the design of packaging machinery motion

submitted by Benjamin Roger Twyman
for the degree of Ph.D.
of the University of Bath
1999

Abstract

The motion of packaging machinery is subject to a wide variety of constraints. These have to be taken into account during the conceptual stages of the design process in order to create a machine that can behave correctly. The constraints apply to both the motion of the machine's end-effector and its joints.

In this thesis the motion constraints that are typically imposed on continuous-processing packaging machinery have been examined, and the critical ones have been identified. It has been shown that they can be grouped into the following categories: task-related constraints, clearance constraints, quality of motion, and commercial interests.

A methodology has been proposed to assist in the design of machinery motion. This involves identifying constraints that apply to a particular problem. The approach has been successfully applied in relation to a number of case studies. A crucial part of the methodology is the ability to identify and manipulate important motion constraints. To this end the idea of parametric timing diagrams has been introduced and demonstrated with the case studies.

The thought processes required to produce the diagrams can help to identify gaps in an engineer's appreciation of the design problem, or to highlight significant features in the required motion. The former is important in order to produce acceptable designs. The latter encourages investigation of the range of performance that can be achieved by a particular design concept under different sets of constraints.

Acknowledgement

I would like to express my thanks to my supervisor Dr Glen Mullineux, and Professor Tony Medland for the tremendous help and advice they have provided whilst I have carried out this work.

The task of completing a thesis also requires having a group of friends and colleagues with whom one can discuss the work, and who can provide general encouragement. The following people fall into one or other or both of these categories and I would like to thank them all very much for their help: Dr John McGarva, Andrew Rentoul, Dr Laurence Kenney, Brian Apps, Dr Richard Hall, Drs Gerry & Helen Agnew and all my colleagues at MDI Ltd.

The support of the DTI/EPSRC LINK programme in the Design of High Speed Machinery, together with the collaborating companies Unilever and CMB Technology is also gratefully acknowledged.

CONTENTS

Chapter 1 - Introduction

The research discussed in this thesis relates to the design of machinery, and in particular is applied to certain problems that arise in the design of packaging machinery. Such machines may be used to carry out a single, relatively simple operation on a product at high speed, such as cutting, folding, joining, or material handling. Of crucial importance to the successful operation of a machine is the motion of the tool (often termed end-effector in the literature) that interacts with the product. This has to be planned or designed with considerable care because the motion required to achieve the operation may be rather complex. If it is not suitable then the machine will not be able to carry out its task properly.

Researchers in engineering design have long been trying to identify a structured design process that can assist designers with the difficulties encountered in tasks such as motion design. Within these, a number of design activities have been identified in an attempt to make approaches to the task of designing mechanisms more systematic. During the conceptual stages of design, mechanism synthesis and analysis is carried out. The synthesis of mechanisms includes two activities: type synthesis and dimensional synthesis. The first of these involves determining which mechanism topology would be most appropriate to achieve the desired motion. Having found a suitable mechanism type, the dimensions of its various parts can be synthesised.

Dimensional synthesis of mechanisms required to pass through a number of precision points and orientations has been a popular area of study. Techniques for this are described in Sandor and Erdman (1984), and interest in the subject continues, with recent additions by Bawab, Sabada, Kinzel and Waldron (1997), Roston and Sturges (1996), and Kim and Kwak (1995).

A factor that can complicate the design process for packaging machinery is whether the machine system being created is to run on a continuous or intermittent basis. When a machine runs with an intermittent motion a conveyor brings a product to rest at a tool station, and the tool then carries out its work. When this has been completed the conveyor starts moving again, and carries the product forward to the next processing area. However, it is increasingly common for packaging machines to be designed to carry out their tasks on products that are still moving on a conveyor belt (Seaward and Vernon, 1994), and such machines are sometimes referred to as having a continuous motion. Systems that move products between conveyors or load them into moving machinery can also be classified in this way. It should be noted that use of the term continuous in this context does not have the same meaning as a motion which is a continuous function of displacement. For example, a motion cycle that contains a dwell period can also have a displacement function which is mathematically continuous, despite the fact that the machine's end-effector stops moving (by definition) during the dwell.

A system designed to run with continuous motion has some important advantages over its intermittent equivalent. It allows higher production rates to be achieved because the cycle time is not limited by the need to decelerate and accelerate the product between processing tools. The absence of these periods of acceleration and deceleration can also enable the continuous motion machine to be more efficient in terms of energy consumption. However, the requirement to carry out the processing task on a moving product can present extra difficulties to the designer. This is because it imposes additional constraints on the end-effector motion. Not only must it be devised in such a way that the end-effector tracks the moving product during the processing step, but it is frequently necessary to ensure that it avoids hitting the next product approaching on the conveyor belt, as it returns to begin the next cycle. These constraints clearly do not exist when the processing step is carried out on a stationary product, as in the case of a machine running with intermittent motion.

Kenney (1993) surveyed the existence and use of commercial and academic software tools for machinery design in British companies. He found that mechanism design software was being used mostly for analytical studies of designs, and dimensioning of components. Its use at the specification and concept stages of the design process was less common. This may have been due to the level of suitability of the available software for this type of work, because he found engineers expressing the wish to have more tools to assist with design synthesis and optimisation. In contrast analytical tools in CAD such as Finite Element Analysis have become more widely incorporated in design processes. Erdman (1995) also noted that analytical techniques were more widely covered by currently available software, and saw the area of type synthesis as being important to develop.

Kenney noted that small and medium sized companies were reporting limited or intermittent demands to carry out machinery design work, and this possibly impinges on the commercial development of suitable software. The author's view is that if companies produce products that move, and they develop them with hardware prototypes, then they would benefit from using some form of mechanism software in addition to the CAD and FE software that has become more commonplace. This is not just because of the often-stated reasons such as the opportunities to save time and money on prototypes, and the commercial advantages of "getting to market faster". It is also because of the additional technical knowledge about a design that can be readily gained through detailed study. This knowledge has commercial value just as the more tangible benefits of reduced costs and development cycle times. If engineers can learn more about how their product works and how it might fail, they ought to be more readily able to make improvements to their future products. This applies to the packaging industry in its requirement to make machines that operate reliably. The development-led design process typical of the industry tends more to result in machines that work well under certain circumstances, but which fail for unknown reasons under a different set of conditions. It seems therefore that there is still plenty of opportunity to research the area of design techniques for use in this industry, and to implement these in software tools that allow more design issues to be addressed before machines are commissioned.

The aim of this research has been to investigate how motion profiles can be created from an examination of the constraints on a machinery design problem, and how these profiles can be manipulated to study various design issues. Six example problems typical of those found in the packaging machinery industry have been studied to achieve this objective. A common theme that links all the case studies is that they all involve the design of continuous motion systems.

The thesis continues with a discussion of research work in motion design. The case studies are used to illustrate the variety of constraints that arise when designing packaging and material handling machines, and a methodology for handling these is proposed. The methodology is tested by application to four of the case studies. It is not considered necessary to treat the other two case studies in further detail because their constraints are of a similar form to the others. For ease of reference the reader should note that figures are grouped together at the end of each chapter, and each figure is printed on a separate page.

Chapter 2 Motion design

In the field of machinery design, good motion design is naturally of great importance. The end-effector of a machine must clearly be designed to move in such a way that it does actually perform the task required of the machine, and considerable effort may be involved in arranging for this to happen. Sometimes it is also necessary to design the motion of the joints of a machine, as well as its end-effector. This is because limits often exist on the range of motions that can be performed by individual actuators, and these limits may change according to the configuration of the moving parts.

The task of motion design is often referred to in the literature, particularly in papers on robotics, as trajectory planning (Rees Jones, Rooney and Fischer, 1984). Another definition of interest here is trajectory tracing. This is the problem of controlling the machine so that it can trace a path that has previously been designed. It arises because machine/controller systems are dynamic systems that have inertia and a limited supply of power, so they may not be able to follow a given motion profile exactly. Although these are closely related problems, this thesis concentrates on motion design, but recognises that the practical problems of trajectory tracing impinge on it, and should not be ignored in the design process.

Much of the recent research work on motion design has been carried out in two areas of application: robotics and high speed machinery. This chapter summarises some of the approaches taken, and important aspects of the subject. These include splitting the motion cycle into a number of segments and specifying values of displacement, or its derivatives, at the boundaries of the segments. Procedures of this type are used as a preliminary step to interpolating a motion over the whole cycle. A large amount of research effort has been carried out on the methods used for interpolation, for example the work of Ge and Ravani (1994a & 1994b), MacCarthy (1988), and Jüttler and Wagner (1996).

2.1 Segmented motion cycles

A common theme in the approaches to motion design is to split the cycle into a number of segments, each one representing a portion of the cycle where some action takes place. For example a processing step in one segment may be followed by a return motion in the next, so that the end-effector is in the correct position to begin the next process or cycle. A suitable function is chosen to interpolate the motion in each segment. Often this entails joining simple functions such as dwells and periods of constant velocity with smooth curves.

This technique is well established for the design of cam profiles, and a number of standard function types, known as the cam laws, have been identified and are commonly used (Chen, 1982). Examples of these are trapezoidal and cycloidal acceleration profiles that provide a dwell-rise-dwell motion. Cam driven systems designed in this way often have a one dimensional output, but the technique is easily extended to design profiles for more dimensions, for example, both the position and orientation of a body.

Of particular relevance to this thesis is recent work by Kirecci (1993), and Mashford (1992). Their ideas are discussed in the next two sections.

2.1.1 Kirecci

Kirecci looked at problems associated with trajectory planning and trajectory tracing for programmable manipulators. In particular he was interested in the effect of different interpolation methods and control schemes on the dynamic performance of the manipulator. The motivation was to try to make programmable machines economically viable in a wider range of applications through improved trajectory planning.

As with the other approaches, he used the design technique of splitting the cycle into a number of segments. The boundaries of the segments were given relevant values of position and its derivatives, and a motion was interpolated between these by choosing a particular type of function in each segment.

He extended the range of interpolation techniques by including a formulation of polynomial that used arbitrary powers (Kirecci and Gilmartin, 1994). Real numbers, rather than integers, were used for the powers, and these were selected by an algorithm that attempted to minimise the distance travelled in each segment. One of the motivations for this was to overcome the problem described as 'meandering', or an unwanted oscillation in the resulting curve.

The techniques were tested with an experimental five bar chain manipulator. End-effector profiles were used as input to the software that controlled the machine, which performed the inverse kinematic solution in order to generate corresponding motion profiles for the mechanism's two cranks. The software also made a check that the end-effector profile was contained within the working envelope of the five bar chain. While the machine was running the operator could switch between different crank motion profiles stored in its memory. A learning control algorithm was used that enabled the software to correct the drive signals within a few cycles in order to trace the desired profile. Cycle time was adjusted by altering the sample time of the control card in the host computer. Experiments were performed at different speeds using a range of profiles. It was found that a motion profile designed with the arbitrary power polynomials could be traced more accurately at slow speeds than one using quintic splines, but was less controllable at higher speeds because of a sharper peak in the jerk profile.

2.1.2 Mashford

Mashford's work looked specifically at the problems associated with designing one degree of freedom cam-driven systems. She created a methodology based on constraint modelling ideas which treated the whole system, from the cam/follower pair, through a transforming linkage, to the end-effector. She considered the motion of the end-effector to be of greatest importance because, as she stated, it is the "raison d'être" of the machine: all other parts exist just to move the end-effector in a suitable way for it to do its work. Other constraints of course place limits on what is possible, and she identified these by looking at relationships between the following three functions.

- the end-effector displacement and the cam follower profile
- the follower profile and the cam angle
- the overall input/output relationship of the system: the end-effector and cam input

The methodology involved ranking constraints according to their hardness, or in other words, how important it is for each one to be satisfied. These were applied to the design in order of decreasing hardness. If, at any stage, the introduction of a constraint resulted in conflict with others, attempts would then be made to try to relax the softest constraint. The process was repeated until all design constraints had been applied and satisfied.

2.2 Measures of quality of motion profiles

To ensure good quality dynamic performance of a machine, it is necessary to ensure that the motion profiles meet a number of basic criteria. These are listed below.

- smoothness
- peak acceleration within limits
- low peak velocity

The smoothness of a motion profile is considered not only in space but also time. In fact, a profile that contains a cusp, although not smooth in space, may be composed of three orthogonal displacement components which are smooth functions of time. The end-effector of a system tracing such a profile would decelerate smoothly on the approach to the cusp point, remain stationary at the point for a while, and then smoothly accelerate away. In such cases the motion can be said to be smooth even though the spatial path is not smooth.

The concept of a smooth motion is also reflected in the form of the higher time derivatives of the motion. Rees Jones et al. (1984) and Kirecci and Gilmartin (1994) consider it sufficient to ensure that the velocity and acceleration profiles are smooth functions. In terms of the polynomial interpolation methods, this requires a displacement curve of at least third order, and fifth order is often used in practice.

The desire to keep a motion profile as smooth as possible is due to the influence it has on the dynamic performance of the machine. Vibrations are undesirable characteristics in high-speed machinery because of the negative long-term effect they have on durability and reliability. They can also interfere with ordinary operation, because of the effect they might have on the position of the end-effector. The design techniques that split cycles into segments have to take smoothness into account by matching derivative values at segment boundaries.

There are contrary opinions on the requirement for smooth motions however. Mashford pointed out that there are reports of machines being run with a discontinuous acceleration function without suffering excess vibration. It is also quite possible that small differences in displacement profiles for motions interpolated by different methods are of more academic than practical interest. This is because the differences may well be 'lost' amongst the tolerances of the manufacturing process when the machine is built.

2.3 Robot trajectory planning

Research in this area has been motivated by the desire to make robots operate at higher speeds, and to handle the problems of clash avoidance at the motion planning stage.

Ozaki and Lin (1996) worked on the problem of finding joint trajectories that would enable a robot arm to move from one position to another, negotiating obstacles on the way. They used B-spline curves to represent each trajectory, and an optimisation scheme to manipulate the curves' control points. At each step of the iterative process, the algorithm would check whether clash had occurred with the obstacles or whether dynamic constraints on the joints were violated. If so the control points would be changed to obtain a new set of trajectories, and the process would be repeated until an acceptable solution was reached.

The problem tackled by Ahrikencheikh, Seireg and Ravani (1994) was one of finding a path from one point in a plane to another that avoids a set of obstacles represented by polygons. The typical application envisaged for this was the design of motion for automatically guided vehicles operating in factories. As well as including obstacle avoidance, the motion was also subjected to kinematic constraints, namely limits on velocity and acceleration. The main aim of the work was to create an algorithm that was very efficient in terms of computation time in order to achieve the tasks of finding a path that conformed to the motion constraints. The resulting motions are also described as being 'optimal', but this is defined in the paper as meaning that the motions pass as closely as possible to the obstacles. The technique developed involves dividing the plane into a set of passages between the obstacles, and constructing a network that represents all possible ways through the passages from the start point to the finish point. Candidate paths are tested, and the shortest one that conforms to the kinematic constraints is returned as the optimum.

An example of minimum time control of robotic manipulators is given by Bobrow, Dubowsky and Gibson (1985). The problem solved is as follows. Given a path for the end-effector to follow, equations of motion for the manipulator, and torque constraints for its joints, the motor torque profiles required to drive the end-effector along the path in the minimum time are found. The strategy is to keep the end-effector accelerating or decelerating at the maximum rate until the whole path has been traversed. The need to decelerate whilst traversing the path is because there may be interim positions where the end-effector must be slowed in order to keep inertial forces within the bounds that can be controlled by the motors. The requirement to bring the end-effector to rest is a practical reason, but is not strictly speaking a requirement for this technique because the initial and final speeds along the path do not have to be zero. At any instant the maximum acceleration is determined by evaluating the range of acceleration possible due to each of the motors, and operating at an acceleration level determined by the least capable motor. This information comes from the transformation of joint angles, velocities, accelerations and torques into functions of displacement, velocity and acceleration along the path. The decision as to where and when to switch between the maximum acceleration to the maximum deceleration is made with reference to a maximum velocity curve on the end-effector's velocity against displacement phase plane. In fact the trajectory is determined so that it just touches the maximum velocity curve.

2.4 Computer environments for motion design

According to Kenney (1993) only a small number of computer systems for motion design have emerged in recent years. Two of these are discussed in this section.

2.4.1 MOTDES/MOTION

This commercial program (Anon. 1996) has been developed, in part, as a result of the research work carried out by Kirecci. It allows the user to construct graphs of an output variable (for example, displacement) against time or another independent variable such as machine angle. This is done by splitting a cycle into segments, and specifying one of a range of functions to describe the motion in each one. There are standard functions to design dwell-rise-dwell motions (for example, cycloidal, modified trapezoidal, and modified sine functions), and dwell-rise-return-dwell motions. Alternatively segments might be described as a period of dwell, constant velocity, constant acceleration, or more generally using a polynomial function and explicitly defined segment boundary values.

2.4.2 RASOR

This software has been developed by Medland and Mullineux as a general tool for solving engineering and geometrical problems. More recent implementations of it are known as CAMFORD and SWORDS. It is described in more detail in chapter 4. Some recent applications of the software include the redesign of a flying guillotine mechanism, a linkage selection program for the tasks of function generation and path matching, and a velocity matching design problem for a transfer mechanism.

The flying guillotine mechanism was used to cut a moving strip of paper into pieces of known length. The constraint modelling system was used to investigate the performance of this, and improvements were made by undertaking a redesign exercise (Medland and Mullineux, 1991). It became apparent during the initial investigation that a better design of output motion (the motion of the guillotine blades) could greatly reduce acceleration levels in the moving parts. A new motion was designed, using line and arc segments, that satisfied two precision point constraints and the velocity matching condition required during the cutting portion of the cycle. This motion profile was used as input to a geometrical model of the linkage, and the application of assembly rules at points along the cycle enabled the pitch curve of the driving cams to be generated. The resulting points on the pitch curves were smoothed by use of a second computer program, and points along these curves were used for all subsequent work. Hence, when the new pitch points were used to drive the geometrical model, the output motion of the blades was no longer exact line and arc segments, but the critical positions in the cycle were found to be still within tolerance.

The design approach taken here involved constructing the desired motion in the xy plane, rather than on separate timing diagrams for the vertical and horizontal components. The timing constraints were relatively simple to calculate by hand, and so it was convenient to work in this way.

The linkage selection applications in the constraint modeller can be used for the tasks of path matching (McGarva, 1994), function generation (McGarva and Mullineux, 1992), and non-uniform motion generation (McGarva, 1995). They are tools for the dimensional synthesis of a variety of linkage types: four-bar, five-bar, and six-bar chains, slider-cranks, and quick return mechanisms. One of the benefits of using these programs is that the mechanisms designed are all driven by constant speed cranks. This can make for relatively simple implementation of the conceptual design.

In the case of path matching, the user first sketches a path using either a closed B-spline curve, or a series of points. If a series of points is used then it is possible to make the system consider the separation of the points as an indication of the relative speed of the end-effector around the path. The program then calculates the complex Fourier coefficients of this target path, and normalises them to remove the effects of scale, rotation, and translation. The normalised coefficients are then compared with those contained in a catalogue file for the particular mechanism type. The catalogue also stores normalised dimensions of the mechanism that corresponds to each set of coefficients. The coefficients are calculated from a parameterised version of the curve. The parameter may represent timing information; by using arc length as the parameter, time independent coefficients are found. Having made comparisons with every entry in the catalogue file, the system then extracts the closest matching mechanisms, and constructs them on screen by applying the inverse normalisation to their dimensions. The designer is therefore presented with a range of candidate mechanisms, and can evaluate them further if necessary. Use of the system has shown that mechanisms returned are often suitable starting points for design optimisation work. This can involve making small alterations to dimensions in order to improve the match to the target path, or to satisfy some additional constraints.

An example of this is the design of a mechanism to transfer goods from one conveyor belt, travelling in one direction at a particular speed, to another, travelling in a perpendicular direction at a slower speed (Medland, Mullineux, Rentoul & Twyman, 1996). This work began by using the path matching program to find a mechanism that approximately matched a triangular path. The four-bar chain selected for subsequent work was chosen because its ground points were in a suitable position. The constraint modelling software's optimisation routine was then used to adjust the link dimensions and mounting points until the two desired conveyor speeds were matched.

The results obtained by using the time dependent Fourier coefficients in the path matching program are a little more crude than for the time independent case. In the majority of cases the shape of the path is not matched as closely as for the time independent version. Also the separation of points on the end-effector locus is only roughly in proportion to the separation of points on the target curve. However, this is an effective means of distinguishing between mechanisms whose return motion is relatively fast as opposed to being relatively slow.

The constraint modeller handles the problems of function generation and non-uniform motion generation in a similar way. However, instead of sketching a path, the designer fits a curve through precision points on a timing diagram. The Fourier analysis, normalisation, and catalogue searching are performed as for the path matching. In this case, the Fourier coefficients obtained are real numbers. Obviously the timing of the end-effector's motion is taken into account by the creation of the timing diagram, and, unlike the case of time dependent path matching, very close matches can be obtained for both displacement and time. Success is however dependent on the mechanism type being investigated, and this is therefore an example of type synthesis being of importance. In practice the catalogue selection is so rapid that within a few minutes the designer can gain the necessary insight to gauge which mechanism type may be more suitable.

A useful application of the program is in investigating the possibilities of replacing a cam-driven device with a linkage mechanism driven at constant speed. This is often of interest to designers because linkages can have more favourable wear and inertia characteristics. There are also currently many cam-driven systems in existence in the packaging industry, and so there are correspondingly plenty of opportunities to make design improvements.

2.5 Conclusion

Research work on motion design has been carried out in the areas of geometric modelling, robotics, and in extensions to the traditional techniques for designing cam-driven systems.

Existing work with the constraint modeller has not concentrated particularly on separating motion design from the design of a mechanism.

With the exception of Mashford, other authors have not discussed in detail the design constraints on motions that are imposed in order for a machine's task to be achieved. For the design techniques involving splitting the machine's cycle into segments, the boundary values between segments tend to be treated as given, and the robotics applications concern the performance of a given machine. However, it is not a trivial task to identify a machine's real motion requirements and to convert these into mathematical expressions of constraint or objective. Having done this a certain amount of creativity is useful in design to investigate how the constraints can be manipulated, and what effect doing this might have on the hardware designed.

Mashford looked at identification and relaxation of constraints, and developed a methodology to apply them in order to design cam-driven systems that satisfy their motion requirements. There is to be scope for extending these ideas in order to investigate how far a machine's performance can be boosted by modification of its motion.

Chapter 3 - Typical constraints on motion

This chapter identifies the typical constraints that are imposed on the design of the motion of packaging machinery. It begins by looking at a number of case study examples, and describes the type of motion constraints that arise in each of these. Some additional categories of constraints are then also discussed.

3.1 Case studies

This section introduces the six case study examples used in the thesis. All but one are derived from design problems encountered by engineers working in the packaging industry. The final one is academic in origin, but illustrates another type of problem and a design principle that can be applied in other cases. The general design requirements associated with each example are described, and the constraints on machine motion imposed by these are discussed.

3.1.1 Case study 1 - filling machine

This example involves the design of a system to fill containers (which may include bottles, jars, or tubs) with liquid products. A schematic of the system is shown in figure 3.1 showing the events that occur during the cycle. The containers move on a conveyor belt at a constant speed, and when they pass into the filling machine, a nozzle moves into position so it can dispense the liquid product. To do this it has to move at the same horizontal speed as the container so that the nozzle remains in the correct position. For certain combinations of product and container type it is also necessary for the nozzle to move down into the container, and move out while the liquid is being dispensed. This can help to avoid splashing inside the container, by keeping the nozzle at an approximately constant height above the liquid surface. However, for reasons of hygiene, the nozzle must not come into contact with the dispensed product.

Having filled one container, the nozzle then has to be moved back against the direction of the conveyor, ready to fill the next container. A constraint that might at this stage be taken for granted, but which proves important when designing a suitable motion, is that the nozzle must not collide with the next container.

The diagram in the figure also includes a sketch of the basic form of the motion profile to be designed for the nozzle (part (e)). This is split into the 3 phases that occur during the cycle: moving down into the container whilst matching the conveyor speed, moving out of the container whilst dispensing the product, and returning to the start to meet the next container.

3.1.2 Case study 2 - ice-cream cutter

This example concerns the design of a device to cut a continuous extrusion of ice-cream into bars. The extrusion takes place vertically downwards, and once cut, the bars fall onto a conveyor belt to be taken on for further processing. Constraints are imposed in this case by the requirements of the cutting process. These are that the cut face of the bar is flat (within a certain tolerance), and that cutting occurs at a known, constant speed (also within a tolerance). Figure 3.2 shows a schematic of this arrangement, together with a sketch of the basic form of the motion expected of the cutting blade. This includes the straight line portion that is needed to meet the processing constraints together with a return.

There are even more possibilities for clash to occur between different parts of the system in this example. For example, on the return stroke the cutting blade must reach the start point of the cycle without hitting any of the following: the nozzle, the advancing face of the extrusion, the product that is falling onto the conveyor, products already on the conveyor, or the conveyor itself.

The design specification also requires the possibility to operate this machine at a number of different combinations of cutting speed and throughput rate. Ranges are specified for these factors, and it must be possible to vary the settings independently. In other words it must be possible to operate at a high cutting speed but low throughput rate and vice versa.

3.1.3 Case study 3 - can handling machine

A design of forming machine for beverage cans consists of a large turret that contains tooling to process blank, unformed cans. The turret has a number of stations around its circumference, and each one contains the same tooling. A large number of stations is required to ensure that production volumes can be as high as possible for a given overall size of machine. The turret revolves at constant speed, and needs one device to load blank cans into it, and another to remove the processed cans. A significant complication arises from the fact that the loading and unloading processes cannot occur instantaneously. This is because tooling on the turret has to rise up to grasp a can before the loading device can let go of it. Consequently, over a certain arc length along the turret's circumference, the loading and unloading devices must track the position of one of the turret's stations. (The reader should note that further reference to this case study in the thesis concerns just the design of the loading device.)

Figure 3.3 illustrates the sequence of events by tracing the path of a can from the point it is picked up by the loading device, around the circumference of the turret where it is processed, and on towards the unloading section. The diagram also shows the arc segment along which the loading device tracks the turret.

3.1.4 Case study 4 - linear conveyor transfer

This case study represents a common class of problems in packaging machinery design: that of transferring products from one conveyor moving in a certain direction, onto another conveyor moving at a different speed in another direction. Figure 3.4 illustrates the basic arrangement. Products are picked up from the first conveyor at position 1, and are carried round to position 2 on the second conveyor belt where they are set down. In order to avoid damaging the product items or risking knocking them over, the transfer device must match the corresponding conveyor speeds at positions 1 and 2. The speeds and directions are therefore constraints on the motion of the transfer device. Often it is not particularly important to specify exactly where on the conveyors the transfer should take place, but spatial constraints imposed by the design of other elements in the system often dictate how much space can be taken up by transfer machinery.

3.1.5 Case study 5 - slitting machine feeder

This case study involves the design of a system to feed sheets of steel into a set of circular rotating blades which cut the sheet into thin strips. Sheets are pushed towards the blades on a dog chain, as illustrated in figure 3.5. To ensure they enter the rotating blade smoothly it is necessary to accelerate them away from the dogs as there is a difference in speed between the dog chain and the tangential speed of the blades. A mechanism to do this has to be designed, and the sketch in the figure shows the basic motion profile that is expected. This includes a small region during which the product is accelerated, followed by a return to the start point. Constraints on the motion are that it must achieve the necessary accelerating action and then lift clear of the product in order to avoid causing any damage. At the end of the cycle it must approach the next sheet in such a way that it matches the speed of the dog chain (so it does not damage the rear edge of the sheet it is about to contact), and also that it does not interfere with the front edge of the following sheet.

3.1.6 Case study 6 - pushing boxes into a labelling machine

The final example also involves the problem of loading products into a machine; in this case stationary boxes are pushed into a machine that fixes labels to them. Although academic in origin, this example is a subtle variation on the problem posed in the previous case study. An illustration is shown in figure 3.6. Boxes are moved into position 1, ready to be pushed along the track into the machine. By choosing a motion for the pushing mechanism that loops over the start position, the next box can be put there whilst the mechanism is on its return stroke. This means the cycle time for loading the labelling machine is only limited by the cycle time for the motion of the pushing mechanism. Constraints on the motion of the pushing mechanism's end-effector include pushing the box over a specified distance and having a horizontal component of velocity of zero at positions 1 and 2. (It is assumed that the box remains in contact with the end-effector even when the mechanism decelerates as it approaches position 2.) During this part of the cycle the path of the end-effector has to clear the track but also has to stay low enough to remain in contact with the rear face of the box. On the return portion the end-effector has to avoid the labelling machine, the conveyor, and the next box waiting at the start position.

3.2 Types of constraint identified in the case studies

It is clear from the above analysis that two distinct types of motion constraint can be identified by studying the basic requirements of a machine. These are the constraints that relate to the specific processing task the machine is to carry out, and constraints that relate to maintaining adequate clearances between moving parts so that clash does not occur.

3.2.1 Task-related constraints

There is a design problem common to each of the case study examples described above. This is essentially one of creating a physical device which has an end-effector motion that tracks the position of a moving product or tool. The task-related constraints allow this product-tracking or tool-tracking action to be achieved. In other words they are derived directly from the overall task required of the machine. Typically they consist of three components.

- A time component. This is determined by how much time is required for the processing event to be carried out. An example from case study 1 is the time needed to dispense the dose of liquid into the container.
- A precision point, or tolerance box component. This is necessary to define a position (perhaps with a tolerance in both space and time) with which to locate the tool-tracking or product-tracking action.
- A velocity matching component which makes the relative velocity between the product and tooling equal or close to zero. In some cases the process can be assumed to take place instantaneously, and the velocity matching is therefore only required at a point in

space. In other cases a non-zero time is needed to complete a process, and therefore the velocity matching has to be maintained over some length of path.

3.2.2 Clearance constraints

These arise from the problem of clash avoidance. It is often the case in machinery design projects that part of a system is already designed, and the task is to create a device to work in conjunction with the existing hardware. This is certainly true for the case studies. For the design of the filling machine and ice-cream cutter, the nozzles, conveyor belts and products already exist. The turret of the can forming machine is already designed, as are the conveyor belts, products, and rotating blades, and labelling device in the other examples.

In some cases it is necessary to consider the possibility of clash with moving objects. Examples of this comes from case study 2, where the ice-cream cutter, on its return stroke, must avoid both the advancing extrusion of ice-cream, and the cut block falling onto the conveyor belt

Given that clash avoidance is a general characteristic of the design tasks posed in these examples, it is obvious that clearances must be taken into account to ensure parts to not interfere with each other when the machines are in operation.

Other types of constraint exist in motion design, and are discussed in the next sections. These are not so obviously identified by looking at a list of a machine's processing requirements or existing design. The first of these relates to the particular design concepts for the physical devices created to produce the motion. In effect they are measures of quality of a motion for the particular type of device, be it a linkage, cam-driven mechanism, or programmable actuator. The second type of constraint is also rather subtle and derives from commercial interests.

3.2.3 Quality of motion and mechanism

These constraints are derived from the physical capabilities of the hardware that creates the motion. There is little point designing a motion if no arrangement of linkages, drives, manipulators or control system is physically capable of reproducing it. As is discussed by Kirecci (1993), consideration has to be given not only to the motion of the end-effector of the system, but also to the motion at the joints of the moving parts of the mechanism or machine. It is possible to create a good quality motion for the end-effector, but then to find this that the joints of the physical system would experience poor quality motions.

Various measures of quality have been applied to motion design in the literature and some are discussed in section 2.2. For example, it is often stated that it is desirable to create smooth displacement, velocity, acceleration, and jerk profiles for the motion, with suitably low values for the peak velocities and accelerations. The motivation behind this in part concerns enhancing characteristics such as reliability, noise and vibration, but also to ensure that the motions are physically achievable.

There are also specific measures of quality for particular types of mechanism. For example, the transmission angle in a linkage mechanism and the pressure angle in a cam-driven system are both required to stay within certain ranges in order to ensure smooth operation, and avoid the risk of jamming. The quality of an output motion might therefore be considered poor if a linkage or cam mechanism tracing the motion possessed unacceptable values for these characteristics.

3.2.4 Commercially-driven objectives

These are constraints that relate to overall machine performance and, if satisfied, can give a machine a competitive advantage in the market place over machines that perform a similar task. For example, there is general interest in being able to reduce cycle times for packaging machinery, as this increases the product throughput rate of the machine. This can have a significant commercial effect upon both the company designing the machine, and its customers. If a company can design a machine to carry out a task more quickly than the designs of its competitors, it can obviously use this fact to win orders and gain a commercial advantage over its competitors. Similarly, companies that use packaging machinery can reduce their operating costs by buying plant that operates at higher speeds.

Other constraints of this type include reliability and power consumption. If plant has greater reliability, then the user of the machine has an even greater advantage over its competitors in terms of reduced operating costs. This comes from less wastage of raw materials and energy, lower maintenance costs, and increased volumes of saleable products created during each shift. Obviously there is a commercial advantage in operating machines that are more energy efficient. Because packaging machinery is run continuously (or nearly so), it is worth striving to reduce power consumption even by a very small percentage. The effect may be negligible over a single cycle, but over the course of a year's production, considerable savings of energy and money can result.

Similarly, machine characteristics such as noise and vibration can be viewed as commercially-driven objectives. However, in terms of design constraints these probably manifest themselves more as limits on levels of acceleration, and these constraints have been included in the previous section. They do however provide more reasons for being interested in designing good quality motions.

Often design problems are couched in such a way that designers are expected to work in a reactive mode, taking specifications from another department and trying to create designs that meet these. For example, production volumes are set, and a machine must be made to achieve these. It is useful if designers can be proactive in negotiating a specification. For example, they ought normally to be in a position to influence a specification by reporting on the capabilities of a design concept, and on the associated compromises, and assumptions. It is therefore useful if the designer considers this kind of constraint and has tools to evaluate different concepts.

3.3 Conflicting constraints

As is usually the case in design, conflict can exist between constraints. It is useful for designers to identify these, and to try to reach a design solution that is in some sense a best compromise between them. The following examples illustrate typical conflicts that can occur in machinery design.

- Attempts to increase product throughput rates, and hence to reduce the cycle time, often compromise the requirements concerning the quality of the resulting motion. For example, velocities and accelerations in the system invariably increase with decreasing cycle time, and these may mean that the motion cannot be traced reliably by a physical system.

- Task-related constraints resulting from a process taking a fixed amount of time also conflict with requirements to reduce overall cycle times. Obviously the cycle cannot be smaller than the processing time. However as cycle times decrease, the proportion of the cycle available for a return stroke is reduced, and if the return stroke length is fixed, the peak velocities and accelerations during the return are therefore increased.

- It is possible to create a motion profile that is suitable for an end-effector to follow but which results in poor transmission angle for the case of a linkage mechanism or pressure angle in the case of a cam-driven mechanism .

3.4 Concurrent design activities

It is debatable whether it is useful or indeed possible to separate entirely the tasks of type and dimensional synthesis from motion design. The question arises because they appear to have been identified as three distinct activities in a design process. One approach to design for example would be to create a motion profile in isolation from any hardware schemes, and to pick a mechanism type on the grounds that it appears a suitable class of device to reproduce the motion in terms of its shape, compactness, speed, tolerances, and location in space. An alternative approach would be to pick a mechanism type and to devise a way of driving it so that the motion requirements are met. In this case the emphasis appears to be on performing the type synthesis in isolation, and the dimensional synthesis takes place simultaneously with the motion design.

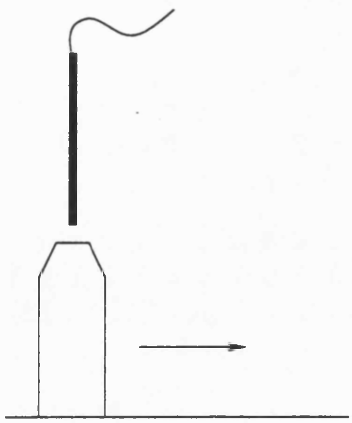
However, it seems more probable that these two approaches represent the extremes in a model of the design process, and that a designer really handles all three tasks (whether consciously or unconsciously) to a greater or lesser degree at the same time. In the first approach it is likely that a motion would be designed with some preconceived notion as to what mechanism would be used to reproduce it. Indeed certain constraints may result purely from the choice of mechanism. In the second approach the type synthesis is performed using information about what is required in conjunction with experience of what can be achieved with different mechanical configurations. If the second approach is being used because the design problem is to make a given machine carry out a new task, then the motion design activity is a separate one, but only because there is no type or dimensional synthesis involved anyway.

3.5 Concluding remarks

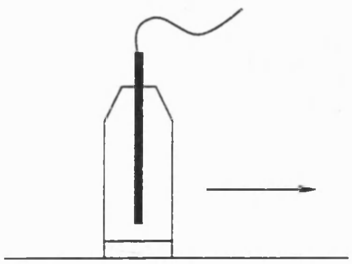
It has been shown in this chapter that a large variety of constraints can be imposed on the design of a machine's motion. These fall into a number of categories.

- task-related constraints
- clearance constraints
- quality of motion
- commercial interests

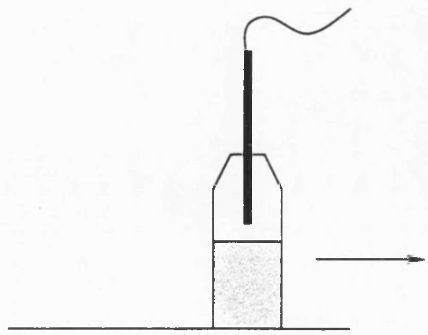
Because so many constraints exist and need to be satisfied, the designer needs some method of handling them. It is proposed in this thesis that a methodology based on identifying constraints and optimising with respect to them can be applied. This is demonstrated by use of a constraint modelling environment that incorporates optimisation techniques to satisfy the constraints and resolve possible conflicts between them.



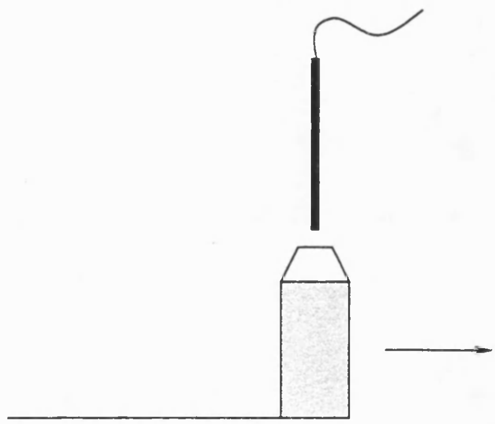
a) Nozzle enters empty bottle



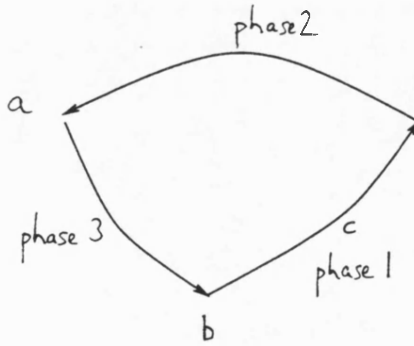
b) Filling begins



c) Nozzle retracts whilst filling



d) Nozzle exits bottle and returns



e) Rough sketch of nozzle locus

Figure 3.1 Schematic of container and nozzle motion

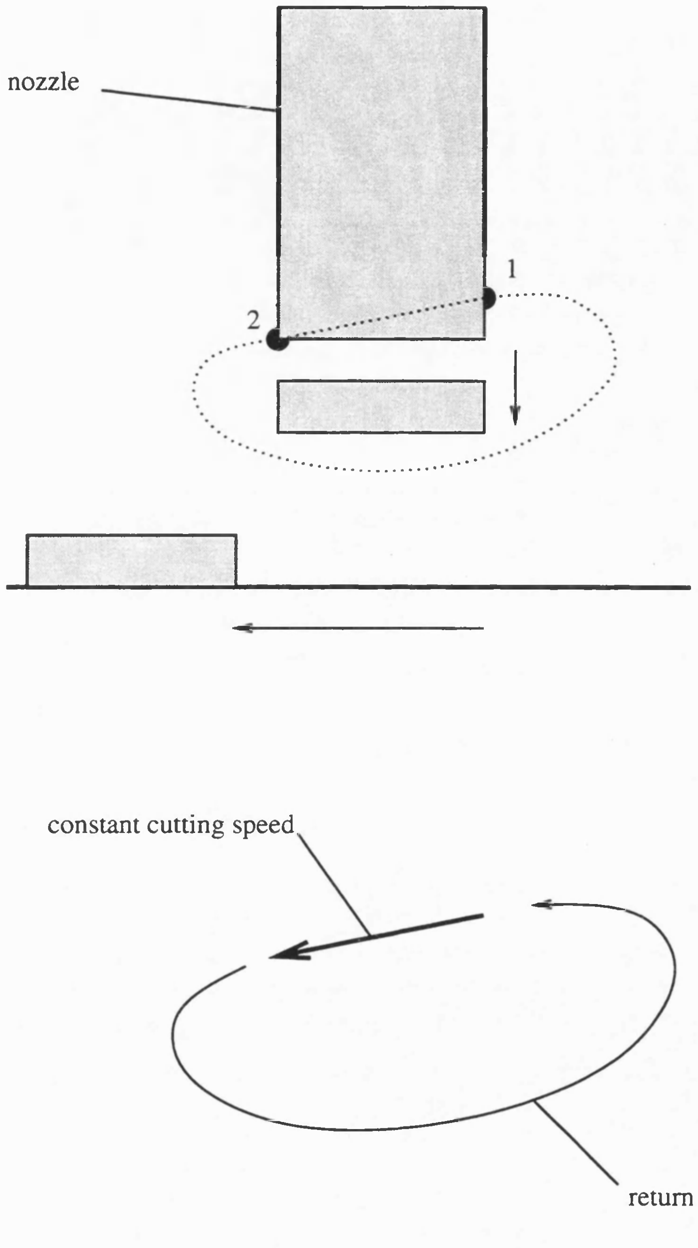


Figure 3.2 Schematic of ice cream cutter path

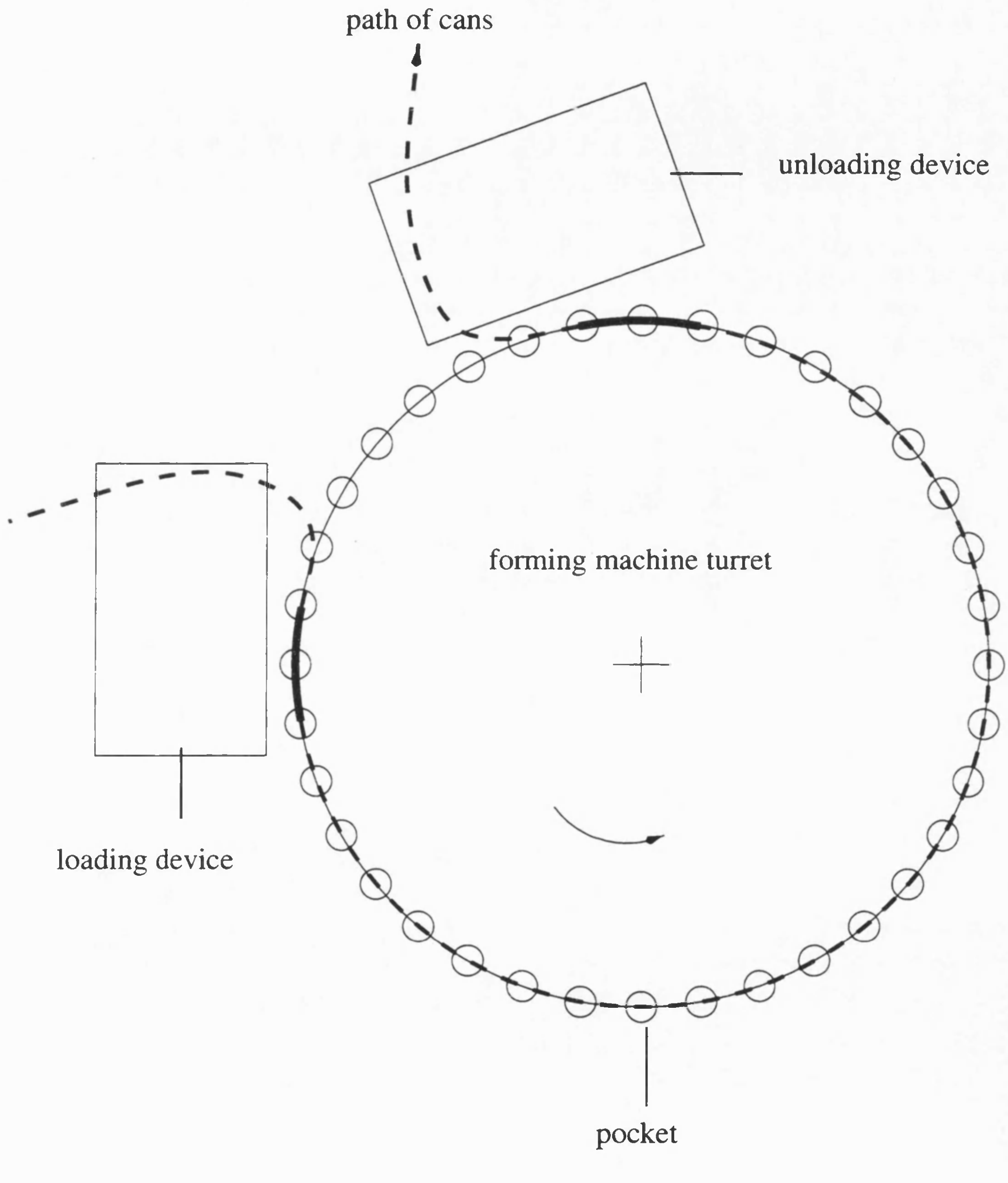


Figure 3.3 Path of cans through the forming machine

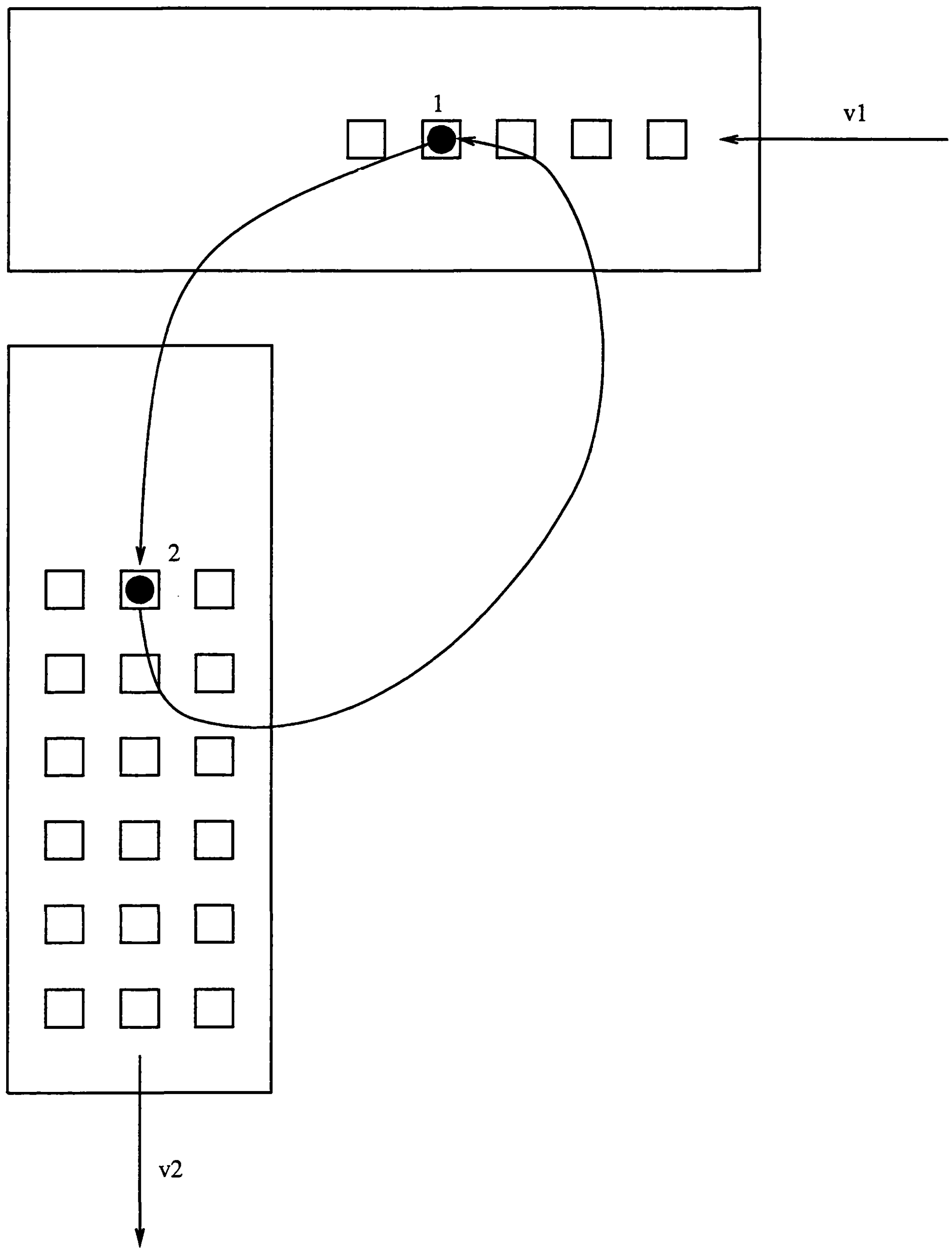
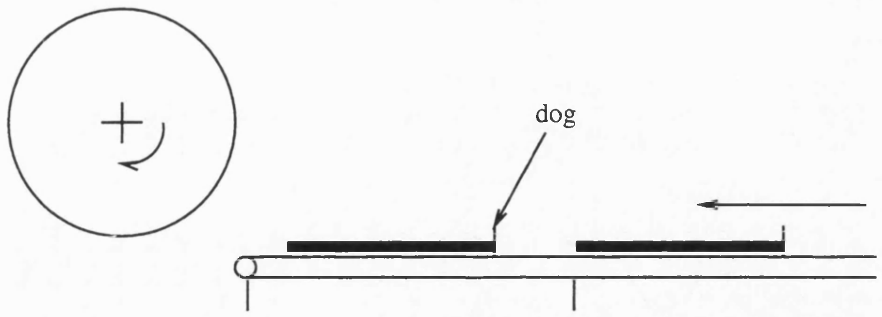
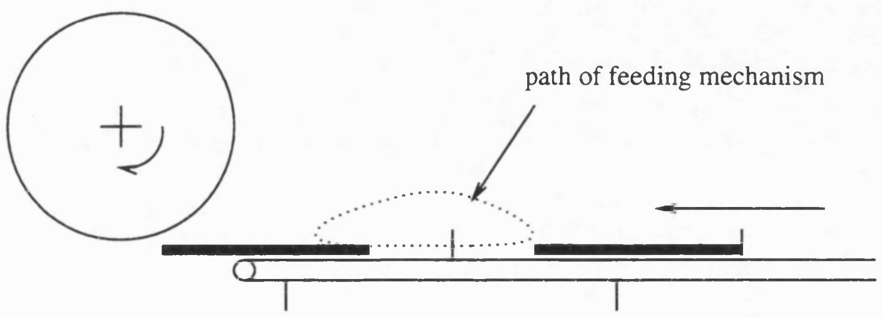


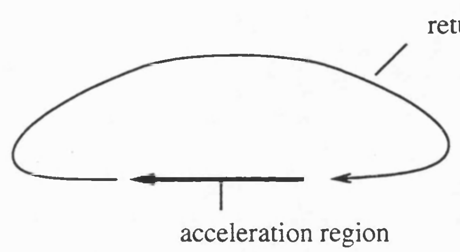
Figure 3.4 Transfer of products between conveyor belts



a) dog chain pushes sheets towards rotating blade



b) sheet accelerated away from dog by feeding mechanism



c) basic shape of feeding mechanism locus

Figure 3.5 Feeding sheet metal into a slitting machine

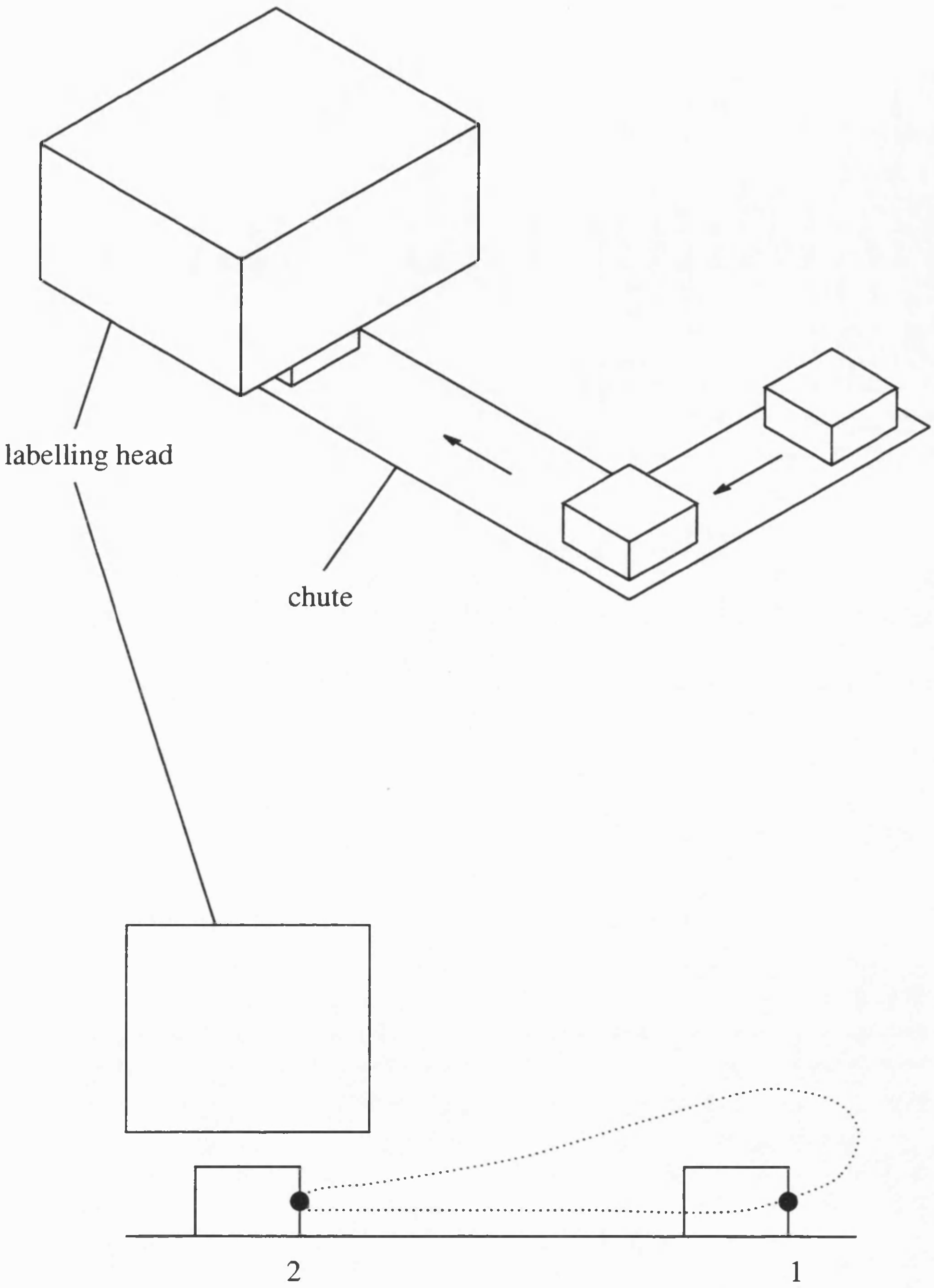


Figure 3.6 Boxes pushed into a labelling machine

Chapter 4 - The constraint modelling software

The constraint modelling software used for this work has been developed over a number of years by Medland and Mullineux, and this has been reported and discussed in papers such as Medland and Mullineux (1991), Medland, Mullineux, Twyman and Rentoul (1995). Its development has been driven by a desire to allow the constraints that bound a design problem to be manipulated within a software system in such a way that designers can gain greater insight into factors that determine how a design will work or fail. In some cases this can lead to satisfactory design solutions, and in others it allows conflicting requirements to be identified and compromises between them to be investigated.

The software provides an environment in which expressions and geometrical entities can be created to represent both the design itself and a set of its requirements or constraints, and then allows these to be manipulated. A simple example of this is the problem of checking for the successful assembly of a four-bar chain mechanism. Lines can be drawn in the graphics part of the software environment to represent the links of the mechanism, and the assembly constraints are created by writing expressions that force the ends of adjoining links to remain together. In order for the system to be able to satisfy these constraints, it must be allowed to adjust certain design variables, and in this case the angles of the coupler and driven links would be suitable ones to use. Having applied the constraints the graphics display will either show a successfully assembled mechanism, or (if the Grashof criterion is not met) a set of links that do not fit together correctly. If the angle of the crank link is then incremented and the assembly constraints applied, the system will attempt to draw the mechanism in its new configuration. Thus, if the process is repeated several times, it is possible to use the system to determine whether the input link of the mechanism can be driven through 360° without the mechanism breaking.

An underlying command language exists in the software to allow the designer to interact with it in order to carry out the type of work described in the last paragraph. Commands can be issued by typing them at a command line, or through macro programs stored in text files. The system parses each command and then interprets and executes it. The language has features common to other programming languages in that variables of certain types can be declared, a set of built-in functions exists, and special purpose functions can be written for use with specific applications. Common file handling functions and the usual programming control structures such as conditional statements (e.g., 'if, then, else' clauses) and repetitive loops (e.g., 'while' loops) are also available, and help make the system a very powerful tool.

Mathematical expressions are constructed to represent design constraints, and these are considered to be true or satisfied when the expression evaluates to zero, or close to zero. In the example of the four-bar chain mechanism given above, the assembly constraints could be expressed as the distance between end points of adjoining links. Thus, when this distance becomes zero the assembly constraints are satisfied. The system uses an optimisation algorithm to handle the constraint expressions, and manipulates design variables in order to try to satisfy the whole set of constraints. When interacting with the system the designer has the option of forcing some variables to be held at given values, but allowing others to be adjusted automatically by the system.

4.1 Design optimisation with the constraint modeller

One of the most useful features of the software is its optimisation capability. It provides an interactive environment in which the designer can pose an optimisation problem, and investigate the effect of changing the objectives, constraints and design variables. A number of factors should be considered by the designer when using the system in this way in order to assess the results obtained. These are listed below.

- formulation of an objective, or objectives.
- selection of constraints.
- formulation of constraints.
- weighting of constraints.
- selection of design variables.
- limits on permissible values for the design variables.
- selection of a set of starting values.

The importance of being free to experiment with these factors during a project comes from the typical nature of 'real' as opposed to 'academic' design problems. In the case of a 'real' design, it is not necessarily the case that any of the above factors can be identified with any degree of certainty at the outset. Often the relative importance of certain factors only becomes apparent after some initial work has been carried out. It might not be obvious at first sight what the most important variables are in terms of the effect on a design's performance. Similarly the initial selection of an objective may well have consequences for the performance or design of other parts of the system, and this may well be something which the designer could not be expected to foresee.

Experience of using the constraint modelling software has led to a number of points about its use in design optimisation. These are discussed in the next four subsections.

- formulation of an objective
- choice of constraints
- choice of design space
- selection of starting values

4.1.1 Formulation of an objective

All the factors listed in the previous section have an important bearing on the success of using optimisation techniques in design. The formulation of an objective has been placed at the top of the list however because of its close link to the meaning of the word 'optimisation' itself, in other words 'achieving the best'. The objective is the term which defines in what sense an 'optimised' design is the best, but it is not a trivial question to determine what this should be for a particular design problem.

Often a single objective is selected for optimisation, such as cost, weight or energy, but this is not necessarily a very realistic approach for certain types of complex products. These are more likely to have a number of conflicting objectives, and the success of a design might be better viewed as how good a balance has been achieved between them. An example of this comes from the design objectives of a suspension system in a passenger car. This has to control the position and orientation of each wheel for the purposes of road-holding and traction, but also has to isolate the car occupants from bumps in the road. These two objectives are seen as conflicting requirements. If one or other of them could be formulated as a single objective in an optimisation problem then an unsatisfactory product would probably result, because the design would be biassed towards just one of the attributes. This example also indicates another difficulty in addressing the question of formulating an objective function for use with an optimisation algorithm. A car is subjected to an enormous range of operating conditions, and these may in themselves present another level of conflict. An expression that may constitute a good measure of quality under one set of operating conditions might result in a design with inferior performance under a different operating regime. It is therefore of great importance to the designer to learn what trade-offs have to be made, and to be able to do some work to evaluate certain compromise solutions.

4.1.2 Choice of constraints

As with the case of identifying a design objective, it is not necessarily an easy task to decide what constraints bound a design problem. One approach is to try to identify all the constraints before doing any computer modelling and then enter them at the same time into the software environment and have the system try to resolve them. However, it is more often the case that certain constraints only become apparent after initial work has been carried out and perhaps some unexpected or unwanted results have been obtained. Whilst gaining this knowledge the designer is also able to learn about the relative importance of certain constraints. This idea formed part of the methodology for cam design created by Mashford (1992). Although she took the approach of trying to formulate all constraint expressions before using the constraint modeller to resolve them, she did advocate running the system repetitively, each time including more constraints, until all constraints are applied. Another technique that allows the designer to observe how the system produces different results is to apply weighting factors to the constraints. This can provide valuable insight which may well be lost if all the constraints are applied at the same time.

4.1.3 Choice of design space

Another way in which design solutions provided by the system can be affected by the designer is by restricting the size of the design space. This can be achieved either by declaring a greater or smaller number of the design variables as ones that can be manipulated by the system, or by bounding the range of values each one can have.

A very large unbounded design space might give a greater variety in results, and for this reason might be considered preferable to using a more restricted one. However, it has the disadvantage that there may be insufficient time available on a design project for the system to search through an enormous design space. The decision to limit its size is therefore another compromise the designer using this type of software should be aware of.

4.1.4 Selection of starting values

The choice of initial value for each of the design variables can sometimes influence the outcome of the optimisation procedure. For example, if an objective function contains several local minima in the design space under consideration, it is possible that a different minimum will be found if the search is started from a different point in the space. This is because the optimisation algorithms tend to find local as opposed to global minima. This may or may not be important to the designer. If time permits, and the designer deems it important to make as thorough a search as possible, then one technique to overcome this problem is to carry out a number of different searches, each one starting at a different point. The outcome of this may be that all or many of the searches 'home in' on the same (or a similar) result. In this case the designer can feel fairly confident that the result is as good as can be achieved for the given conditions of objectives, constraints, and design space.

Alternatively, there may be a greater variation in results, and this may indicate a type of sensitivity of the design to the design variables.

4.2 Conclusion

The constraint modelling software provides a useful environment in which to investigate how all the factors described above can affect the generation of design solutions for a particular problem. It allows the user to combine any objective or constraint expressions interactively to gain the insight needed to examine various design issues. Given the range of typical constraints on packaging machinery motion described in the previous chapter, this is very useful tool with which to tackle the case study examples in this thesis.

Chapter 5 - Proposed methodology

This chapter proposes a methodology that compliments the existing constraint-based techniques for the design of packaging machinery. The basis of the technique is to use knowledge of the processes, constraints and general requirements of the machine to create parametric timing diagrams that model the motion requirements. It is only necessary to include sufficient constraints on the diagrams to allow valid motions to be created.

A number of activities need to be performed, and these are listed below. Their interaction is shown in figure 5.1.

- sketching the basic form of the motion
- type synthesis
- construction of the parametric timing diagrams
- optimisation of the combination of hardware and designed motion

The tasks of sketching, type synthesis, parametric programming, creation of timing diagrams, and optimisation have been included before in design processes. However, a methodology proposed in this chapter brings them together and allows the timing diagrams to be constructed in a parametric form. This permits constraint-based techniques to be employed to highlight significant features of the design problem, and produce optimal designs.

The methodology described here initially identifies motion requirements, and then proceeds by invoking software to handle these. The discussion here assumes that the constraint modelling software is available, although support could equally well be provided by other computer-based tools, or even be handled manually. However, it is envisaged that computer support might be beneficial in many cases, because this makes the testing of very large numbers of alternatives feasible.

5.1 Identifying the basic form of the motion

This activity involves considering information about the overall requirements of the machine to be designed, and using it to sketch the basic form to be expected for the motion. Sources of information include not only formal specifications, but also existing designs and knowledge. The typical questions that need to be addressed concern what product or tool has to be moved around to perform the task, and in what directions and orientations it should move. Often what emerges is that the motion cycle can be split into two types of part.

- periods that are quite rigidly defined or governed by the process requirements.
- periods that join these together, often forming a 'return' in preparation for the next cycle.

Furthermore it is common for the number of these periods to be low. Typically just one or two operations are carried out by the machine during the cycle. For the purposes of sketching the expected form of the motion it is important to differentiate between these parts, because naturally there is a large difference in the amount of freedom allowed between the two types. A simple method, for example, is to use solid lines to represent the regions where the motion is tightly defined, and dashed lines to represent the general shape of the return. At this stage the designer can give thought to how much freedom there is to vary the form of the return motion. It becomes clear after going through a design iteration that the range of feasible shapes for the return can depend heavily on the conceptual design of the hardware used to implement the motion. For this reason the sketching and type synthesis activities naturally take place simultaneously.

5.2 Type synthesis

This stage involves choosing the topology of the device to carry out the machine's task. Often the generation of ideas and the decisions made are reliant on the designer's experience, although some researchers have tried to formalise the activity and create tools to assist in it (such as mechanism atlases and graph representations mentioned in Erdman, 1995). The following questions may also need to be addressed at this stage.

- Is the motion design problem one of function generation, path-matching, or rigid body guidance?
- What sort of forces need to be applied by the machine to carry out the task?
- What type of technologies are suitable or available, for example electric motors or hydraulic actuators?
- Is the motion basically rotary, linear, or a combination of both?
- What degree of accuracy is required?

The choice is usually between cam-driven linkages, uniformly or non-uniformly driven linkage mechanisms, servo systems, or geared systems.

In this thesis it is suggested that when using the constraint modelling approach to machine design it is sensible to investigate the potential of a uniformly-driven linkage first of all. Experience has shown that the constraint modeller's mechanism selection application can bring good results with relatively little effort (as described in section 2.4.2). It allows the designer a high degree of creativity because it does not require rigorous specification of requirements before presenting a range of alternatives. In effect it leaves it to the designer to make the initial judgements as to whether a particular linkage type might satisfy the motion constraints. The quantitative assessment by the computer can come later. The system can also can be effective in spurring ideas as to how to go about tackling the problem by the fact that so many motion shapes can be tried in a very short amount of time.

However, there are machine design problems where uniformly-driven linkages do not provide an appropriate solution. Often this is because the motion requirements are too stringent and the constraints cannot be relaxed sufficiently. Examples of this are listed below.

- motions that contain a large number of constraints
- constraints that cover a large proportion of the cycle
- tasks that require very high positional accuracy
- tasks that require well-controlled constant speed motion over an appreciable length of path
- machines that require a wide range reconfiguration (for example, to cope with product variation)

In cases such as these there might be more potential in devising a cam or programmed servo-driven system, and timing diagrams are naturally very useful tools for designing these.

5.3 Construction of the parametric timing diagrams

The construction of the parametric timing diagrams requires two activities to be performed. One of these is paper-based, the other is most usefully carried out with a computer, and this is discussed here with reference to the constraint modeller. Later chapters provide the case study examples.

A useful starting point, before writing a macro program for the constraint modeller, is to develop the sketches produced in the first stage of the methodology. This can help clarify in the designer's mind how best to represent the motion constraints. The following list contains the typical activities to be carried out at this stage.

1. Split the motion into orthogonal components.
2. Select an origin for the co-ordinate frame.
3. Split the cycle into its processes, events, and phases.
4. Identify boundaries between the processes.
5. Decide whether time values for the boundaries or events are fixed by some requirement, or whether arbitrary selections can be or need to be made.
6. Sketch what is known about displacement, velocity, and acceleration requirements in each direction at these times. Are instantaneous values or simple functions (for example, linear or sinusoidal) required? Should values be fixed, or made arbitrary?
7. Consider whether the arbitrary choices should be given lower and/or upper bounds. This might represent some tolerance or clearance condition.

Having carried out the exercise described above, it is a more simple task to generate the macro that manipulates the constraints and interpolates a motion over the whole cycle. The following steps are involved.

1. Insert precision points on the relevant axes to represent instantaneous displacement, velocity, acceleration or jerk constraints. Parametric programming should be used to allow changes to the coordinates to be made simply. Such changes can be made manually by the user, or automatically by the system.
2. Decide what geometric entities to use to interpolate the motion between the precision points. Typical examples are to use line segments joined together by open free-form curves, or to use a single 'closed' free-form curve. If the former is used then an extra step, such as use of another precision point, must be taken to ensure continuity of the motion at the start and end of the cycle.
3. Perform curve fitting to obtain an instance of the motion.
4. Experiment with curve fitting parameters in order to get an appropriate curve. In the case of B-spline segments this can include varying the number of control points, the number of knots and the knot spacings. The constraint modeller's implementation of B-splines is in terms of knot differences rather than the absolute knot values (McGarva and Mullineux, 1994). This step is needed because curve fitting techniques can sometimes give unexpected and unwanted results. Typical of these is oscillation of the curve between precision points.

It should be explained that the term 'precision point' is used here simply to mean a point on the graph through which a curve or function is made to pass. It does not imply that the point's coordinates are in any sense the 'best' or even that it is desirable to have the motion pass through them. The precision point is just a variable geometric entity used in the curve or function fitting. A valid and optimal motion can only be generated if its parameter values are properly constrained.

The parametric representation used here makes it very simple to generate new instances of the motion. This might be necessary for two reasons. The first is to design a motion for a new configuration of the basic design; for example, to account for a different product or set of operating conditions. This entails making changes to certain 'fixed' values. The second reason is that the initial choice of any 'arbitrary' precision points generally results in non-optimal motions being generated. By encoding a suitable objective function and running the optimisation algorithm, the constraint modeller can improve the generated motion.

5.4 Optimising the combination of hardware and motion

The optimisation of machine motion is yet another example of the trade-offs existing in a design problem. In this case they arise because it is often necessary to reach a compromise between a 'good quality' motion and a 'good quality' mechanism. For this reason, the final stage in the methodology concerns trying to achieve an optimum combination of both the motion, and the hardware that produces it.

The typical sequence of events for each trial is as follows.

1. Optimise the timing diagrams to smooth out the effects of any arbitrary variables.
2. Evaluate the x and y displacement diagrams at equal time steps.
3. Combine the resulting (x,y) pairs on the xy plane.

4. Test the effect this motion has on the performance characteristics of the hardware (for example, actuator velocities, or cam pressure angles). If necessary use optimisation to obtain the best geometry for this instance of the motion.
5. Make a change to the motion, by resetting certain precision points or introducing new ones, and repeat the first four steps.

This can become a very lengthy process, especially when optimisation is required to obtain an acceptable geometry for the hardware. In effect there are three optimisation processes required. Two of these (optimising the diagrams to smooth the motion, and optimising the mechanism geometry) are performed for each function evaluation of the third (the optimisation of the combination). Because this can become such a large problem it may, in some cases, be necessary to carry out some of the changes manually rather than with the optimisation algorithm. For example, the designer may choose to investigate the potential of just two or three instances of a motion profile, and just use the system to optimise the machine geometry. These instances should be chosen carefully by the designer so there is confidence that a sufficiently wide search of the design space is being made. Also, by limiting the investigation to just a handful of candidate profiles, it is possible for the designer to make comparisons between the resulting designs manually.

It is not strictly necessary to include every constraint by means of precision points on the timing diagrams. In certain cases the formulation of an objective to satisfy one constraint (for example, minimising acceleration), may have the fortuitous effect of also satisfying another (for example, a clearance constraint). The procedures of optimising the motions should be seen as iterative, starting with a simple description of what might be regarded the most important constraints. If, on inspection of the graphical results, this turns out to be insufficient to satisfy all the constraints, it may be necessary to introduce additional precision points to the diagrams, or error functions to the optimisation in order to make it do so.

The following chapters show how this methodology has been successfully applied to representative examples of the case study design problems introduced in chapter 3. They show that a range of design issues can be addressed by experimentation with suitably constructed parametric timing diagrams.

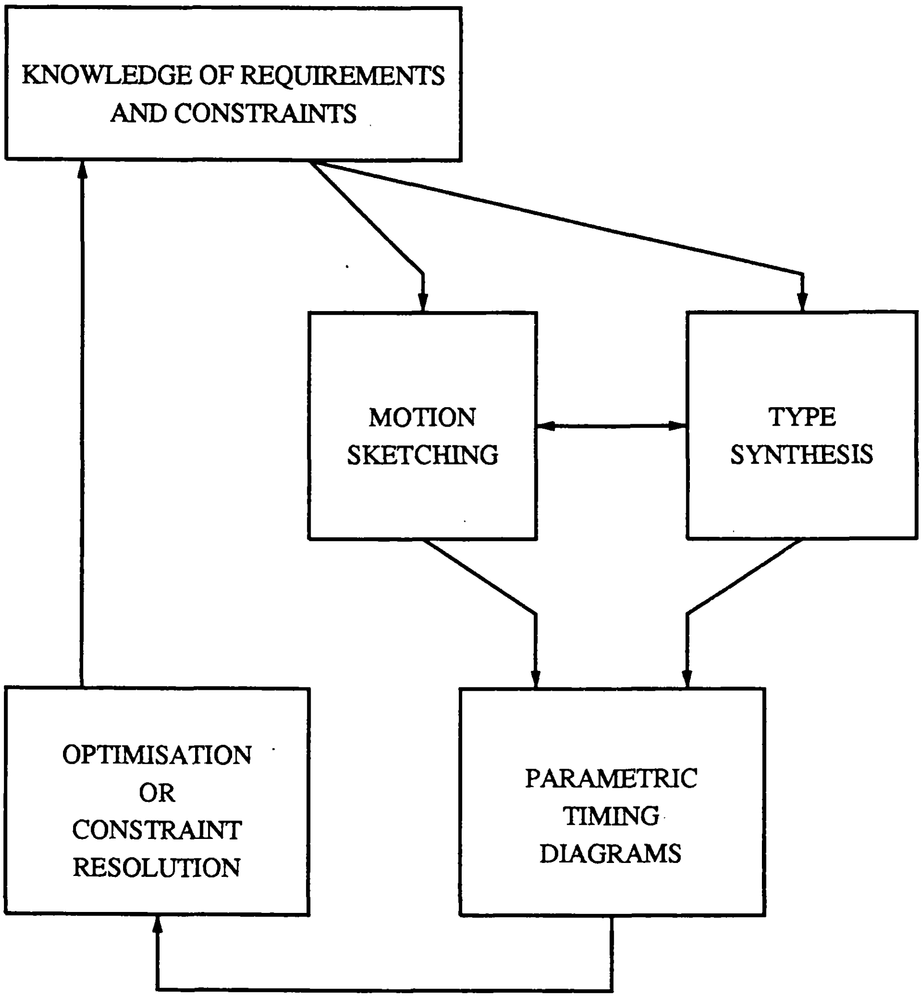


Figure 5.1 Block diagram of the methodology

Chapter 6 Case Study 1 - filling machine

The first case study concerns the design of a machine to dispense liquid products into containers moving at constant speed on a conveyor belt. The interest here is not in analysing the fluid flows in the system, but in identifying the motion constraints imposed on the filling nozzle, and using these to construct suitable motion profiles for the machine. The machine must be sufficiently versatile for the same basic configuration to package different product types into a variety of container types. The liquids themselves might vary in viscosity, leading predominantly to increased filling times, and the containers might vary considerably in size and shape, examples being bottles, jars, and tubs. This section describes how the methodology proposed in chapter 5 has been used to address design issues that arise from the problem.

6.1 Concept selection

It is considered at the outset that the most feasible way of implementing the range of motions required of the machine is to use a pair of orthogonal, programmed, linear drives. These would guide the filling nozzle around a planar path to perform the task at hand, one providing the horizontal motion, the other driving the vertical motion. This would entail mounting one of the axes on the other. For example, the filling nozzle might be mounted on the vertical drive, and the resulting assembly would be mounted onto the horizontal drive. It is assumed that the structure of the machine can be made stiff enough to avoid deflections due to reactions to the fluid flow, or indeed to other dynamic loads. An advantage of this concept is that the nozzle keeps the same orientation throughout the cycle, without the need for some extra linkage or actuator to control its orientation. The design problem here is one of rigid body guidance rather than just path generation.

An arrangement such as this makes it natural to carry out the motion design in the two orthogonal directions, using the parametric timing diagrams to cater for changes in product or container. Furthermore, the resulting motion profiles can be used as input to computer-based CAD or dynamics models of the system, allowing simulation of the machine's performance. This provides some visual feedback to the designer to help verify the suitability of the designed motions before applying them to prototype hardware. In practice the designed profiles would be used as reference inputs to the control system driving the linear actuators.

It seems unlikely that linkage mechanisms might provide an acceptable solution to this design problem for two reasons. These are the requirements for variations in path resulting from the variety of operating conditions, and the need to maintain the nozzle's orientation through a cycle. The first of these might, however, be considered less of a drawback if only a small number of product combinations are to be handled by the machine. Nevertheless the assumption is made in the rest of this chapter that the filling nozzles are to be guided by the linear drives.

6.2 Construction of the parametric timing diagrams

The schematic of the nozzle's motion shown in figure 3.1 splits the cycle into three phases.

- Phase 1 - the liquid is dispensed as the filling nozzle moves forward.
- Phase 2 - the filling nozzle returns to meet the next container.
- Phase 3 - the filling nozzle moves down into the next container.

Furthermore the start and end of each phase can be considered as notable events in the cycle. Identification of these makes it relatively straightforward to insert precision point constraints on the horizontal and vertical timing diagrams (the x-t and y-t planes). These are shown in figure 6.1. The origin of the xy space is taken to be the position of the nozzle at the start of the cycle (for $x = 0$), and the surface of the conveyor (for $y = 0$). The start of the cycle is taken to be the instant at which liquid begins to be dispensed. In the following discussion, the time and displacement values at a particular event are referred to using the following convention: event 1 has coordinates $t1$, $x1$, and $y1$, event 2 occurs at $t2$, and so on.

6.2.1 Motion constraints in the horizontal direction

The horizontal motion of the filling nozzle is mostly influenced by the need to track the conveyor belt speed during phases 1 and 3 of the cycle. This is due to the requirement to fill containers while they are still moving through the system. If the conveyor belt speed is well controlled and can be assumed constant, then the horizontal velocity of the nozzle must also be designed to be constant during these phases. This gives rise to the straight line segments sketched in figure 6.1. A complication arises from the need for the nozzle to successfully clear the container before travelling back against the direction of the conveyor belt. For certain types of product the nozzle is still inside the neck of the container when the flow of liquid ceases. In these cases the duration of the velocity matching must be extended by a short amount to give the nozzle time to move far enough vertically to clear the container. This extension into the return phase is represented on the timing diagram by an arbitrary value $t3$.

6.2.2 Motion constraints in the vertical direction.

In the vertical direction the motion of the filling nozzle is constrained by the need to track the height of fluid in the container during filling, and to provide adequate clearance between the nozzle and the container in the later parts of the cycle. This requirement can be expressed by including a series of precision points on the *y-t* plane (see figure 6.1).

The precision points that give the height tracking motion can be derived from the flow characteristics of the pumping system. The relevant flow data are conveniently obtained from a graph of volume delivered against time. These are compared against a graph of volume against container height in order to obtain a series of points representing the height of fluid in the container at a number of discrete times. An example of this is shown in figure 6.2. The flow data can be obtained from experimental or analytical studies. The graph of container height against volume can be derived by 'slicing' a solid model of the enclosed volume of the container at discrete height intervals. Having derived the fluid height against time relationship, it is sensible to add an offset to the height values. This creates a clearance between the tip of the nozzle and the surface of the fluid. It provides a safety margin to account for splashing and the fact that in practice the fluid surface being formed is not entirely flat.

Finally the height of the container is used to insert precision points at times $t3$ and $t4$. The objective of this is to ensure the nozzle traces a path that enters and exits the container without touching the sides. The y values at times $t3$ and $t4$ are selected arbitrarily, but they must be constrained to be no less than the height of the container.

6.2.3 Selection of parameter values

In order to create an instance of the motion, the values for a number of system parameters need to be specified. These are the overall cycle time, the pitch of containers on the conveyor belt, and the filling parameters.

- Cycle time is determined by the required throughput rate of the machine or the factory.
- The pitch of the containers on the conveyor belt is determined by the design of the fixture that holds them steady during the filling operation.
- The filling parameters include the filling time and the flow characteristics of the pumping system. These are determined by the combination of product type and container type under consideration. Experience and empirical data are generally used to determine a minimum amount of time needed to fill a certain container with a particular volume of a liquid. This is limited by the hygiene and economic implications of splashing or spilling the liquid product. In other words, if the flow is too fast then the liquid might splash onto the nozzle, which may well be unacceptable from a hygiene point of view. If it is so torrential that liquid is spilt outside the container then the loss of product is obviously bad for economy.

Once these values have been selected, all other precision point values can be derived, and the points can be inserted on the computerised form of the diagrams. This includes the precision points for the vertical displacement diagram, and the clearance points needed to avoid clash between the nozzle and the container. Having inserted these points on the diagrams it is then possible for the computer program to fit curves through them to interpolate the motion over the whole cycle.

6.2.4 Curve fitting

At this stage all the elements are in place to allow curves to be fitted through the constraint diagrams. A decision has to be made as to the most appropriate way of doing this. The choices include joining together free form open curves with line, arc or sinusoidal segments, or fitting a closed curve through the entire cycle. As indicated in chapter 5, the decision is influenced by the form of the constraints in the case at hand.

Here there are relatively few constraints on the horizontal motion, but great accuracy is required for the velocity matching. For this reason a line segment is chosen for phases 1 and 3, and an open curve is used to join them together. Obviously the use of the line segments guarantees the accuracy of velocity values interpolated between times *t1* and *t3,* and between *t4* and the end of the cycle. The return phase is created with an open curve segment. This is convenient because in this case it is simple to specify boundary conditions that ensure a smooth transition between adjacent line and curve segments. Because the speed is constant over phases 1 and 3, the acceleration and jerk values at the boundary times *t3* and *t4* are both zero. Hence the curve is fitted with precision points placed at coordinates (*t3*,0), and (*t4*,0) on the acceleration and jerk axes. The total number of constraints imposed on the curve fitting is therefore eight: four at each boundary. This means the open curve must be created with a minimum of eight subsegments joined by knots. The fifth degree spline formulation used for the displacement curve requires the last five knots to have a knot difference value of zero in order to generate an open curve. The knot difference values for the first three knots are spread equally across the interval *t3*-*t4* for simplicity.

The vertical motion is treated differently because such a large proportion of the cycle contains precision points. In fact a closed curve is fitted over the whole cycle. Again a fifth degree spline is used for the displacement, as this results in smooth velocity and acceleration curves made up from quartic and cubic polynomial segments respectively. One knot has been inserted 'under' each precision point, as this has been found to overcome the problem of the unwanted oscillations in the interpolated curve. This phenomenon can be a problem particularly at the start and end of phase 1 (see figure 6.3). Another strategy used to overcome the problem is to choose a sinusoidal rather than linear spacing for the independent variable of the precision points. This allows more precision points to be placed at either end of the segment, and fewer in the middle. Clearly the choice is influenced by the form of the function, and some experimentation is required to obtain a reliable method for the case at hand.

6.3 Design issues investigated with the timing diagrams

It is possible to evaluate motions for a wide range of operating scenarios by generating different instances of these timing diagrams. This is achieved by changing certain parameter values, such as container type, cycle time, filling time or container pitch. Furthermore, the constraint modeller's optimisation features can be used to obtain optimal motions under each operating condition. Construction of the curves also allows certain design issues to be addressed which are not necessarily clear at the outset. Examples of these include examining the importance of container pitch, or determining whether gains can be made by optimising the path during the transition between phases 1 and 2.

Summary of constraints

X direction

- track the conveyor speed during phases 1 and 3.
- extend speed matching into phase 2 to allow nozzle to clear bottle after flow stops.

Y direction

- fluid height tracked during phase 1.
- nozzle above bottle height before returning in phase 2 and entering in phase 3.

Variations

- vary t3 and t4 to investigate characteristics of the motion.

6.3.1 Effect of variations in container type

The first example of a motion profile created using these constraint-based timing diagrams is for a system that fills one litre bottles with water. The required system parameters, as described in section 6.2.3 (*'Selection of parameter values')*, have the following values.

- cycle time, 6 seconds
- filling time, 5 seconds
- flow/time relationships as plotted in figure 6.2
- bottle pitch, 105mm (1.5 times the bottle diameter)

It is notable that the production requirements of cycle time and filling time place a stringent time constraint on the return phase of the motion. Only a very small proportion of the cycle can be taken up by the return.

The conveyor belt speed required for this set of conditions is 17.5mm/s, and hence the horizontal displacement of the nozzle at end of the filling phase (*x2*) is 87.5mm. The velocity matching is extended by 5 degrees of the machine cycle to allow the nozzle to lift out of a bottle before it returns for the next cycle. Because the clearance requirement is handled in this way, the precision point (*t3,y3*), as described in section 6.2.2, is ignored. Initially the 1 second interval available for phases 2 and 3 is split equally so *t4* is given an initial value of 5.5 seconds. The clearance at time *t4* cannot be ignored however. The height of the nozzle at this time point is constrained by setting *y4* to a clearance value of 10mm above the height of the bottle. This ensures that the nozzle is moving forward at the correct speed as it enters the bottle. Two different bottle types are considered here: one with relatively straight sides, the other with a long, thin neck. Height constraints for these bottles are interpolated over phase 1 from the flow data given in figure 6.2.

The constraint resolver's optimisation routines are then used to adjust the value of *t4* with the objective of reducing the peak resultant acceleration. Clearly, given that in this case there is just one design variable, it would be simple to perform the optimisation manually. This would entail testing the whole range of allowable values for *t4* (between *t3* and the end of the cycle), each time fitting the curves and recording the peak of the resultant acceleration. The value to be selected is obviously then the one that gives the smallest peak acceleration. Whichever method is chosen, the optimum value for *t4* is found to be at 5.58 seconds, and this gives a peak acceleration of $5090 mm/s^2$. The timing diagrams and path that result from this set of conditions are plotted in figure 6.4.

Results for the more irregularly shaped bottle are similar in terms of the optimum found for *t4*, 5.58 seconds, and the peak acceleration, $5080\ mm/s^2$. The shape of the path is different however, and is shown in figure 6.5. This is due to the kink in the vertical displacement constraints that results from the abrupt change in cross section between the body and neck of the bottle.

6.3.2 Effect of variations in fluid type

Changes in fluid type result in modification of the flow characteristics and hence also the function to be traced during phase 1 of the cycle. It is also likely that the time required for filling will be different for different fluids, and this can have an effect on the design of the nozzle motion. If it is decreased, and the cycle time and other factors remain the same, then a greater proportion of the cycle can be devoted to the return phase. This in turn results in a lower peak acceleration in the horizontal direction.

6.3.3 Effect of variations in container pitch

The pitch of containers on the conveyor belt also has an effect on the acceleration experienced by the filling nozzle as it returns to complete a cycle. This is illustrated in figure 6.6. The solid line shows the horizontal motion for a machine with a reference value for the container pitch. If a larger pitch than the reference is selected, then the speed of the conveyor belt has to be increased accordingly. This also requires a corresponding increase in the horizontal speed of the nozzle during phases 1 and 3 of the cycle, so that the nozzle can track the containers. This is shown by the dashed line in the figure. The filling time remains fixed and so the nozzle travels further during phase 1. If the vertical motion profile and overall cycle time also remain the same then a fixed amount of time is available for the return phase. This leads to a greater acceleration in the horizontal direction for the system with the larger container pitch. The conclusion drawn from this is that the pitch should be selected to be as small as possible in order to minimise acceleration. The limiting value is determined not only by the diameter of the container, but also the dimensions of the fixture used to hold it in place on the conveyor belt. This is the rationale behind the choice of pitch in the first example of one and a half times the container diameter.

6.3.4 Optimising the transition between phases 1 and 2

In the first example (*section 6.3.1*), the value of $t3$, the extension of the velocity match into phase 2, was fixed for simplicity, and an open curve was fitted between points $(t3,x3)$ and $(t4,x4)$. In fact the need for the extension was only recognised when a motion profile created without one was used to animate a solid model of the machine. It was observed that the nozzle began its return too soon, and by chance only just avoided hitting the side of the bottle neck. The extension of 5 degrees of machine angle was deemed to be a suitable amount to ensure the nozzle lifted out of the bottle before travelling back to the start of the cycle. This is another example of the iterative nature of design. The importance of certain constraints only appears as the design work progresses, and strategies for dealing with the overall problem can be modified accordingly. In this case the next stage in the use of the timing diagrams is to include extra precision points and degrees of freedom into the motion model in order to account for this clearance constraint. Having done this, the optimisation algorithm can be put to work again to see if any further improvements can be made. This involves designing the motion profile to allow the nozzle to return as close to the side of the bottle neck as the designer wishes to allow. It is illustrated by the schematic in figure 6.7b. In effect it 'rounds off' the transition between phases 1 and 2 of the motion.

In order to carry out this task it is necessary to add precision points to both the horizontal and vertical displacement diagrams. Also the open curve fitted on the horizontal axis is adjusted to start at *t2* instead of *t3*. Moreover to account for the clearance constraint on exiting the bottle, the curve must be made to pass through a precision point at (*t3*,*x3*). It is worth restating at this point that event 3 is the instant at which the nozzle lifts clear of the container. By this definition then, *x3* must be constrained to lie within the neck of the container. In effect it is subject to the following tolerance.

$$x3 = (t3 * conveyor\ speed) \pm (radius\ of\ container - radius\ of\ nozzle)$$

Having defined event 3 in this way it is also necessary to force the closed curve on the vertical displacement axis to pass through a precision point at $(t3,y3)$, where $y3$ is a height just greater than the container height.

With the above adjustments made to the timing diagrams, the optimisation process is used to alter the values of $t3$, $t4$, and $x3$. However, this time an extra objective is included. As well as minimising peak resultant acceleration, the peak horizontal acceleration is also required to be minimised. The reason for choosing this as an objective is that the horizontal axis has the greatest inertia, and therefore lower acceleration in this direction would result in lower forces and correspondingly lower energy requirements. It is not sufficient to specify minimum horizontal acceleration alone however, because a trade-off still has to be obtained between the levels of acceleration in the two directions.

Results obtained from optimising the diagrams in this way are shown in figure 6.8. Here the path can be seen to rise to a greater height on the return, but this allows the compromise between levels of acceleration in the two directions to be achieved.

6.3.5 Relaxing the conveyor-tracking constraint

This is a variation on the previous study, and involves allowing the horizontal speed of the nozzle to be lower than the conveyor belt speed. The speed is designed so that the horizontal clearance between the nozzle and container sides is 'traversed' by the nozzle during the filling phase (see figure 6.9). The theoretical maximum average nozzle speed over the filling phase is derived from the gap length and filling time.

nozzle speed = conveyor speed - ((container diameter - nozzle diameter) / t2)

This allows the whole of the clearance to be traversed during the filling phase. The motivation behind making this adjustment is the same as in the previous examples, in other words to drive the levels of acceleration as low as possible. Obviously the greatest scope for achieving reductions comes from product combinations where the nozzle width or radius is considerably smaller than that of the container. Acceleration is reduced because a smaller change in speed is required over the set time interval of the return phase.

6.3.6 Relaxing the fluid-tracking constraint

There is potential to smooth the vertical motion of the nozzle by relaxing the fluid-tracking constraint. This would mean allowing the gap between the nozzle and the surface of the fluid forming in the container to vary. It is therefore a strategy that can be applied when detailed information about fluid flows, and their effects on surface formation, are known. A tolerance band might be defined to indicate a range of acceptable clearances at different stages in the filling process that would still allow control of splashing.

Two methods could be adopted to achieve the smoothing. The first involves implementing each vertical displacement precision point in the timing diagram as a variable, and bounding them so that they cannot be set at heights beneath the fluid surface. The optimisation algorithm could then be used to adjust their values so that acceleration in the vertical direction is reduced. The disadvantage of this is that a very large number of design variables would have to be manipulated by the system. In practice it is likely that the thirty precision points used to cover the filling phase would be too many for the currently implemented algorithm in the constraint modeller to deal with. A better method would be to model the filling phase height constraints with a simpler function, described by a small number of variables and with known derivative values at each end. The vertical motion over phases 2 and 3 would then be interpolated with an open curve, as in the case of the horizontal motion.

6.3.7 Removing the fluid tracking constraint

It is possible to create one dimensional motions with this system by ignoring the fluid tracking constraint altogether. This would mean designing a motion profile for the nozzle that remains at the same height above the containers at all times, but tracks the container on the conveyor belt. The advantage of this is that the return phase can begin as soon as the fluid stops flowing. There is no need to consider the possibility of clash between the nozzle and the containers.

6.4 Concluding remarks

This case study has demonstrated the following points regarding the use of constraint-based parametric timing diagrams in motion design.

At a simple level it is an example that naturally requires many different product configurations to be considered, and hence the usefulness of the parametric representation is illustrated. Changes to certain key system parameters are all that is required to generate a new instance of the generic motion.

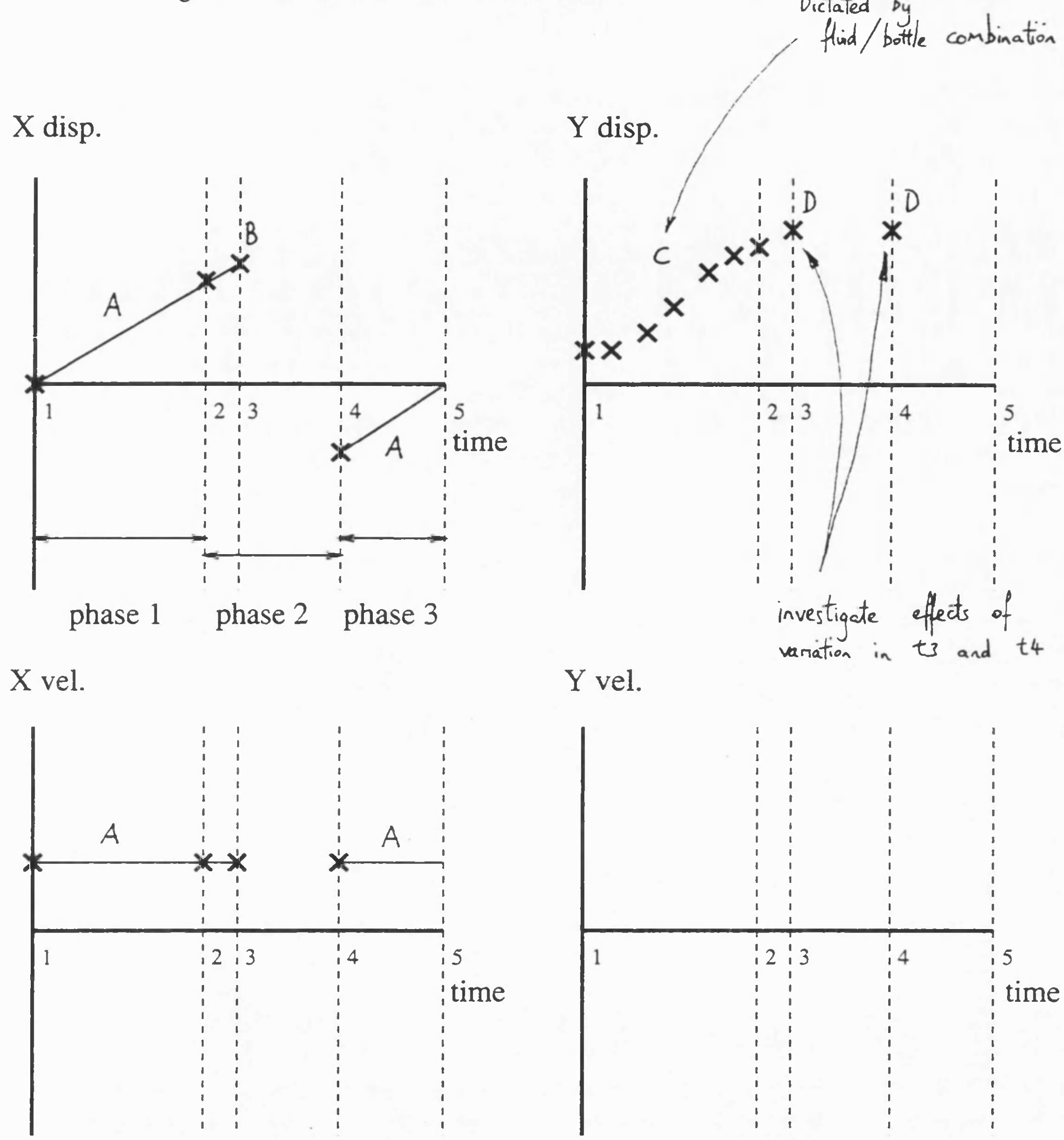
Rather more subtly, the methodology has helped to identify small details in the requirements of the motion. An example of this is the extension of the velocity matching into the return phase to allow the nozzle time to lift out of the container. The methodology has helped because it forces the designer to consider how much freedom there is to vary the motion at different points in the cycle. This is reflected in the computer implementation by the creation of variable precision points whose values are manipulated by optimisation.

In this particular example, optimisation has been performed to limit the peak resultant acceleration experienced by the filling nozzle. It has also been carried out to weight the motion towards lower acceleration in the horizontal direction. This could help reduce energy consumption as the horizontal actuator carries the vertical axis and nozzle, and therefore has the greater inertia. The system requirements and constraints have been handled in a variety of ways, to provide strategies for achieving reductions in acceleration.

Further parameter studies could be carried out using the timing diagrams. For example, they could be used to examine the trade-off between cycle time and acceleration levels. In effect this means investigating the effect of changing the proportion of the cycle spent on each phase. This might become important in the case of short, wide containers (such as margarine tubs) which, because of a large nozzle area, only require short filling times. The width of such containers leads to higher conveyor velocities. This means that, to keep accelerations down, the return phase must become a greater proportion of the cycle. Obviously this requires an increase in overall cycle time.

The motions designed by the system have been used to animate solid models of the linear actuators, bottles, and conveyor belt, thus allowing the operation of the machine to be simulated. (Wireframe graphic output from the constraint modeller is shown in figures 6.10 and 6.11 as an example.) This provides a visual check that ensures the motions are acceptable before they are implemented in the hardware. They can also be used as the reference input to the actual controllers used in the physical system.

- Phase 1 - liquid is dispensed.
- Phase 2 - filling nozzle returns to meet next container.
- Phase 3 - filling nozzle moves down into next container.



Summary of constraints

X direction

A track the conveyor speed during phases 1 and 3.
B extend speed matching into phase 2 to allow nozzle to clear bottle after flow stops.

Y direction

C fluid height tracked during phase 1.
D nozzle above bottle height before returning in phase 2 and entering in phase 3.

Figure 6.1 Parametric timing diagrams

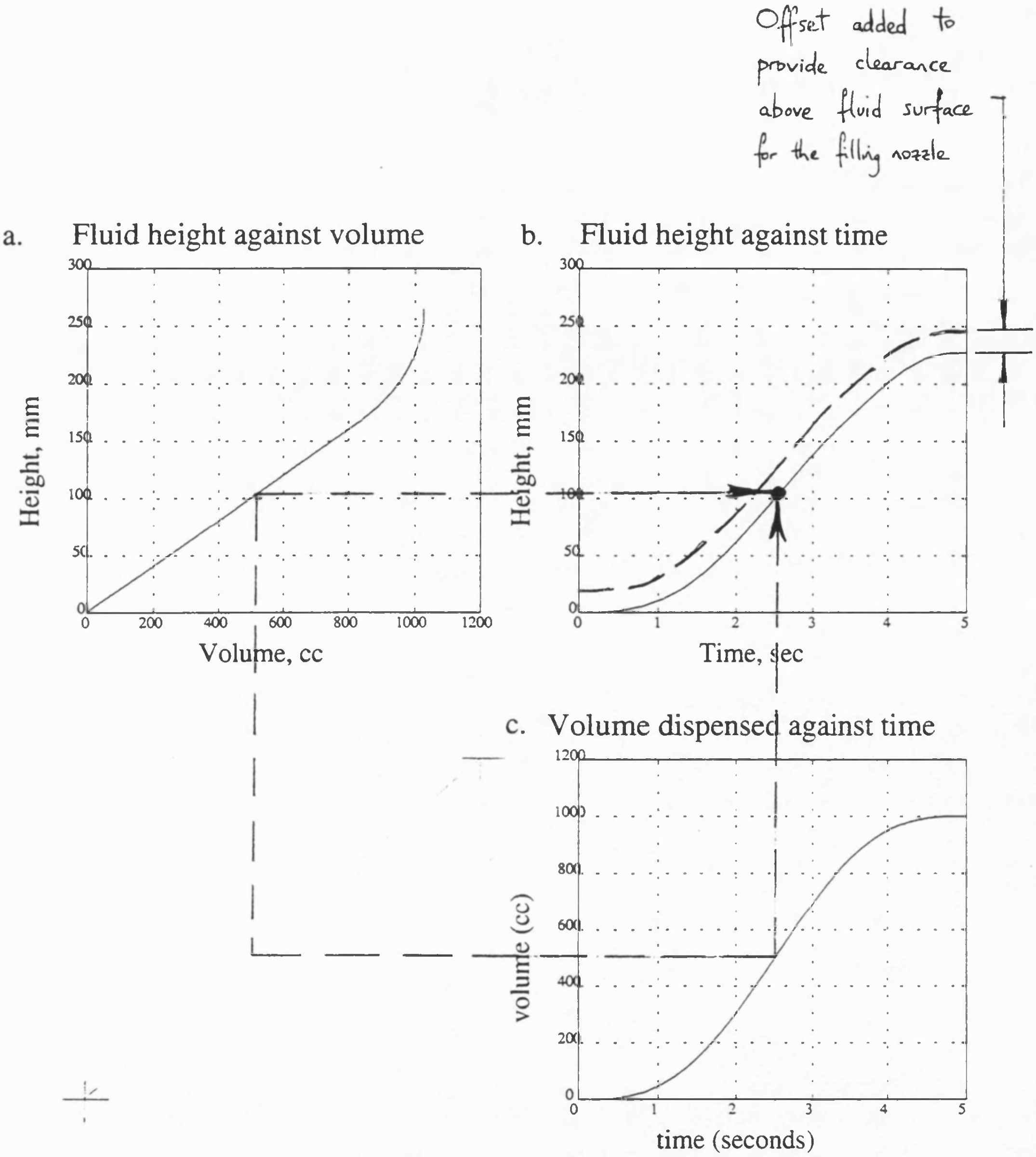


Figure 6.2 Flow characteristic

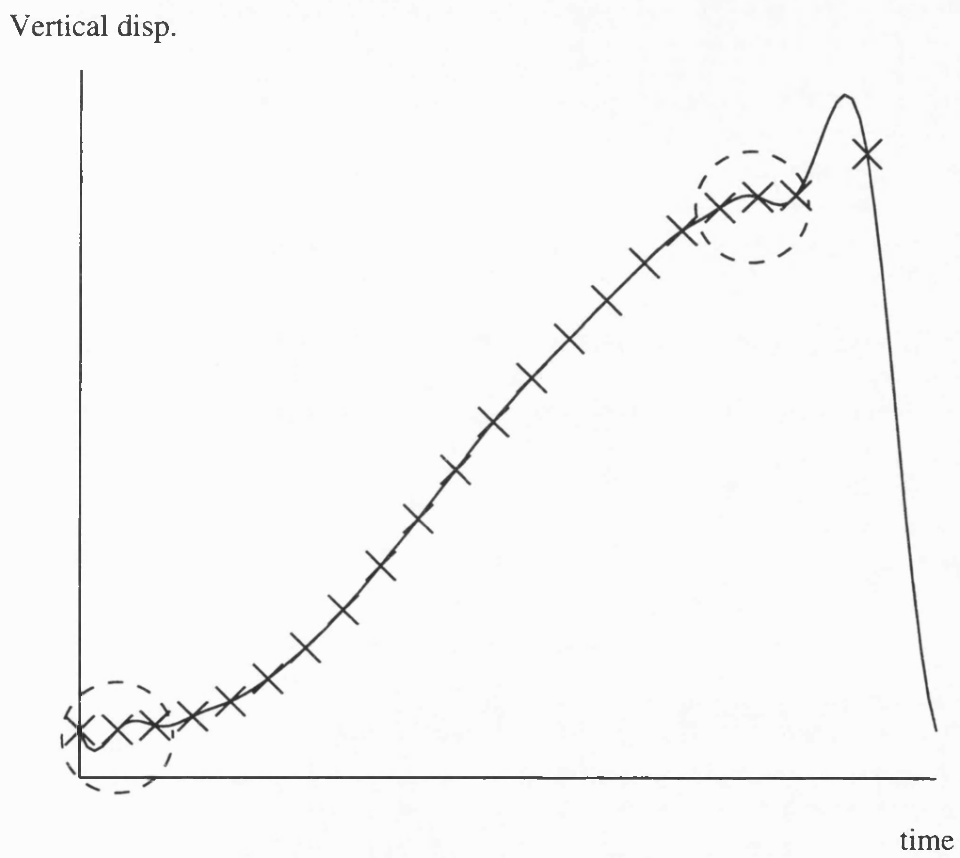
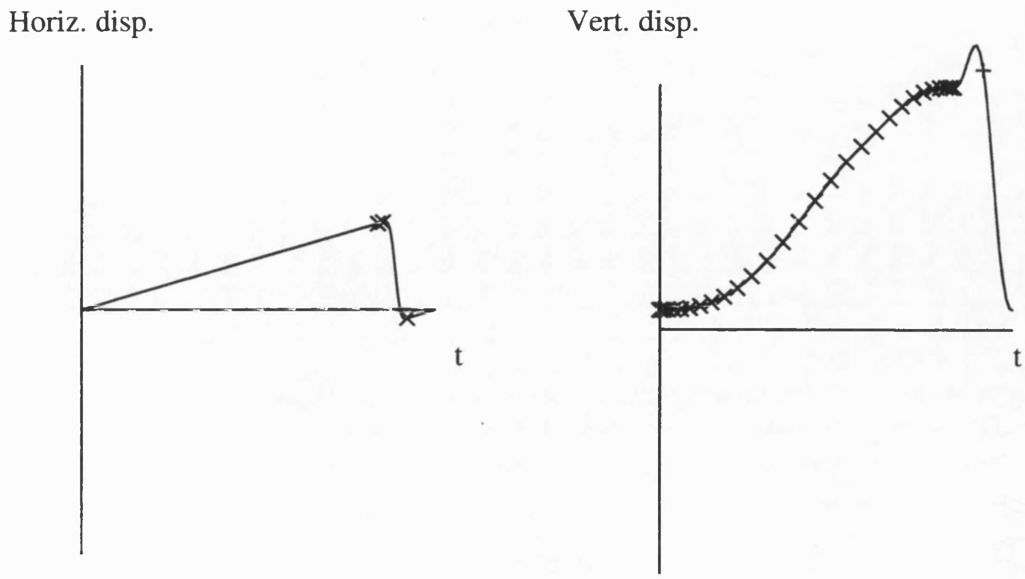


Figure 6.3 Unwanted oscillations in the interpolated curve



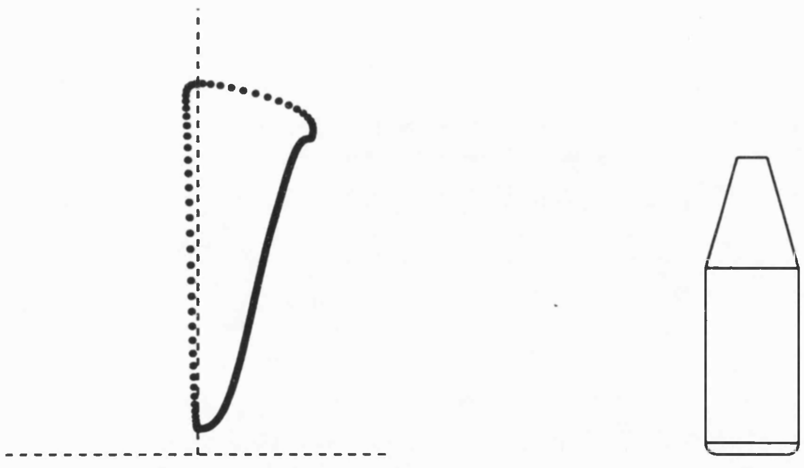


Figure 6.4 Motion profile to fill the straight-sided bottle

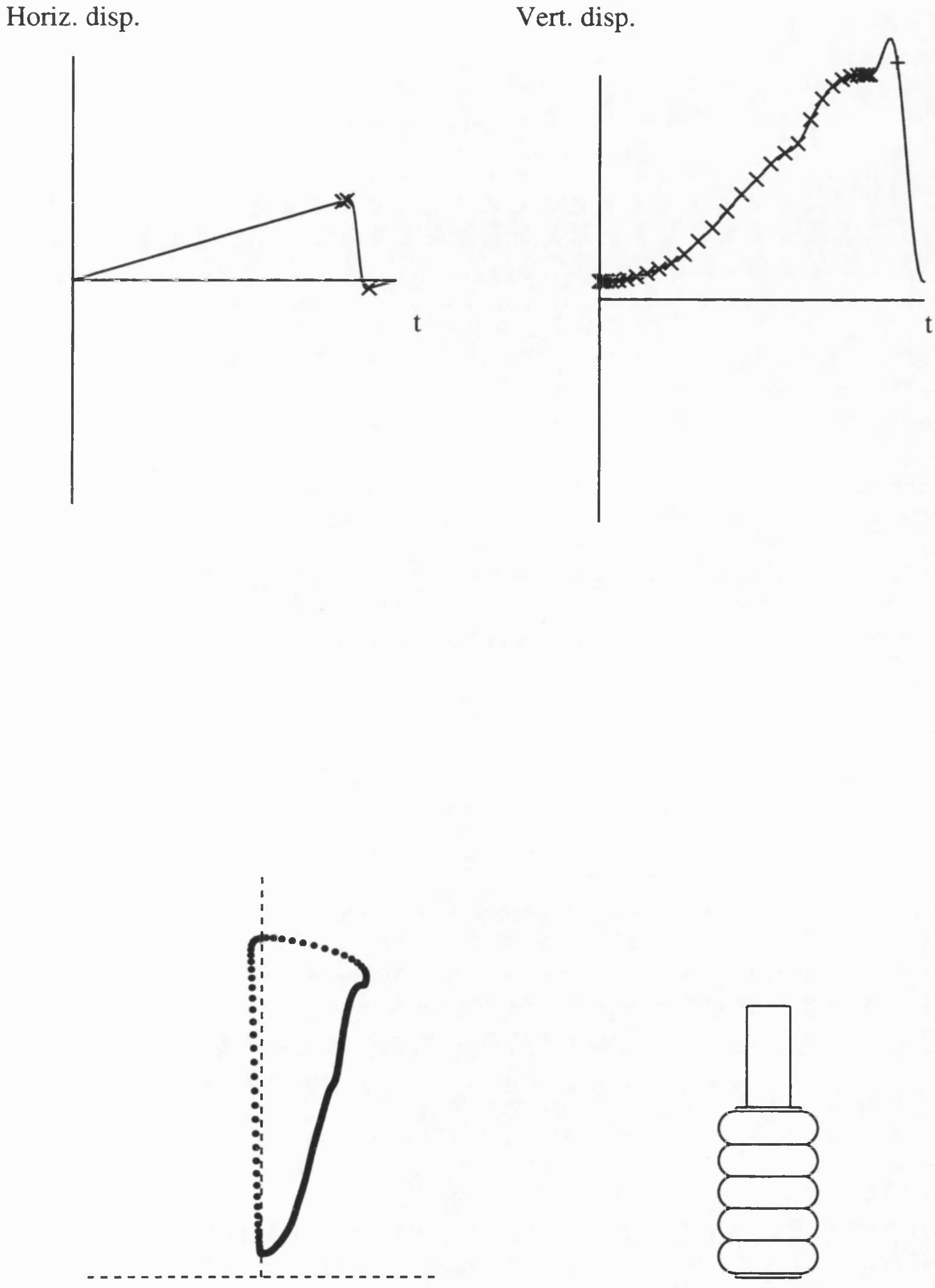


Figure 6.5 Motion profile to fill the alternative bottle

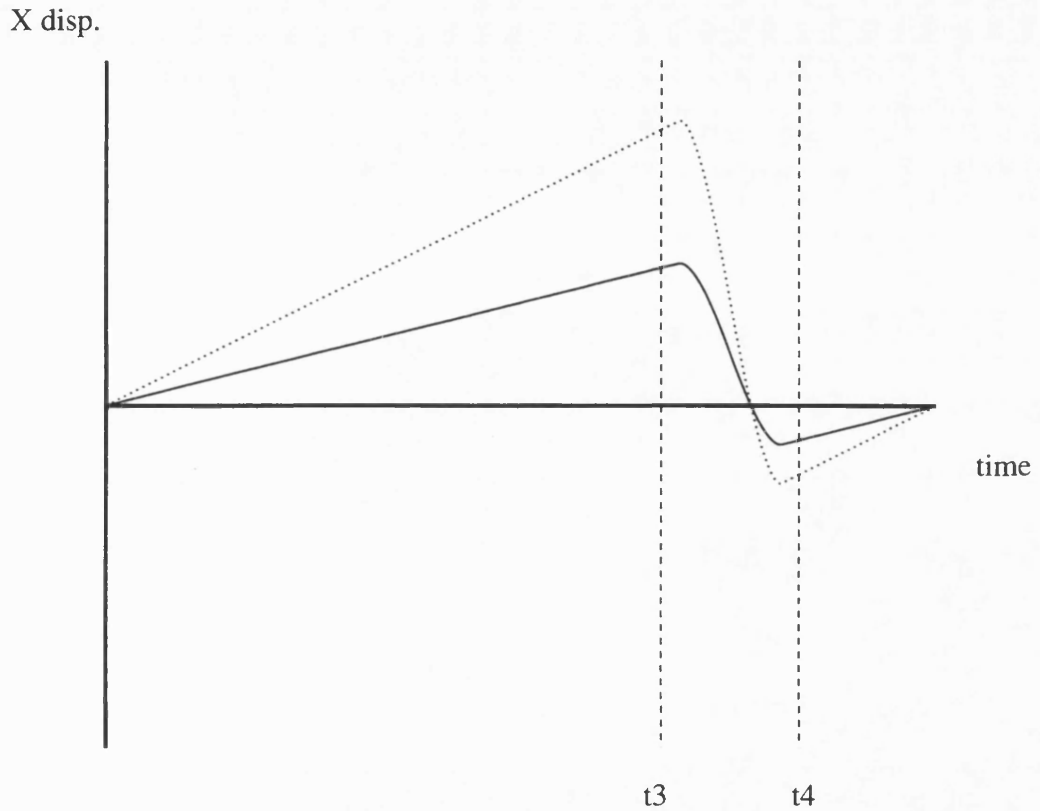
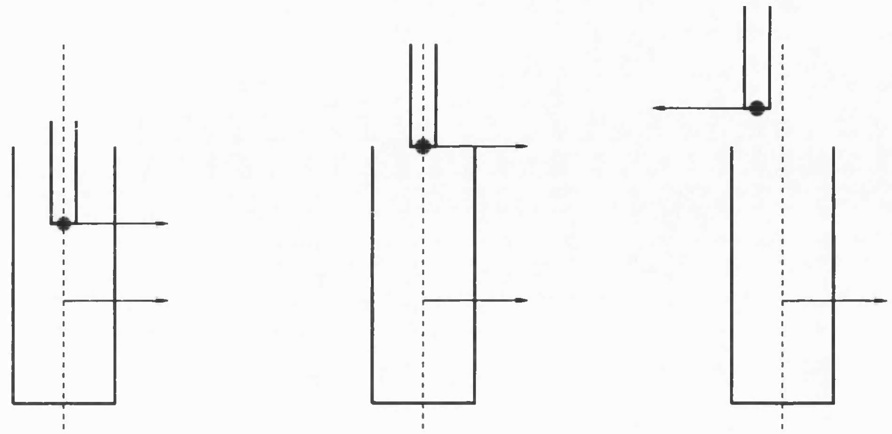


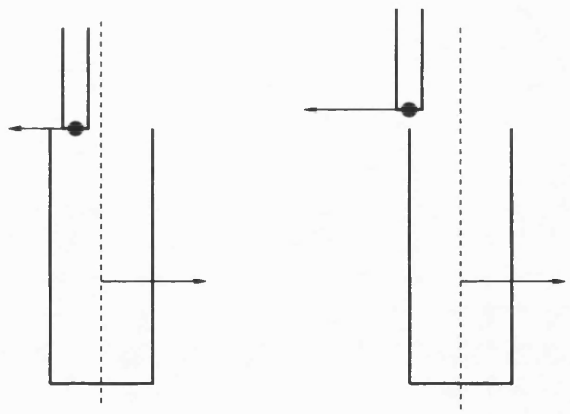
Figure 6.6 Effect of variation in container pitch

a. Initial model: nozzle tracks conveyor until bottle neck is cleared



during phase 1 — at time t3 — during return

b. Optimised model: nozzle just clears bottle neck



during phase 1 — at time t3 — during return

Note: arrows represent horizontal components of velocity

Figure 6.7 Transition between phase 1 and phase 2

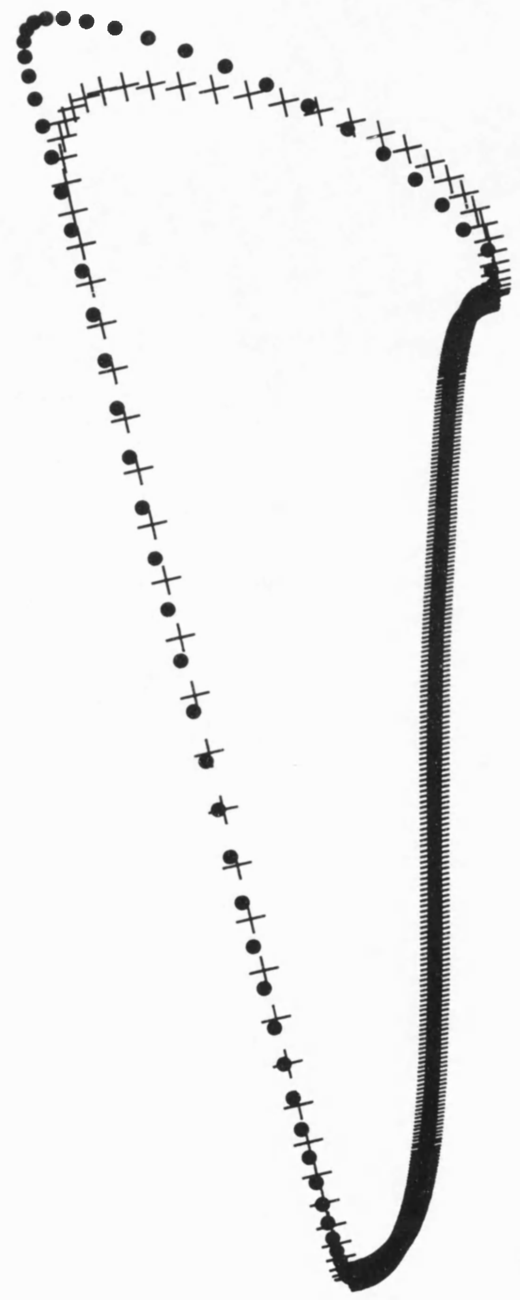
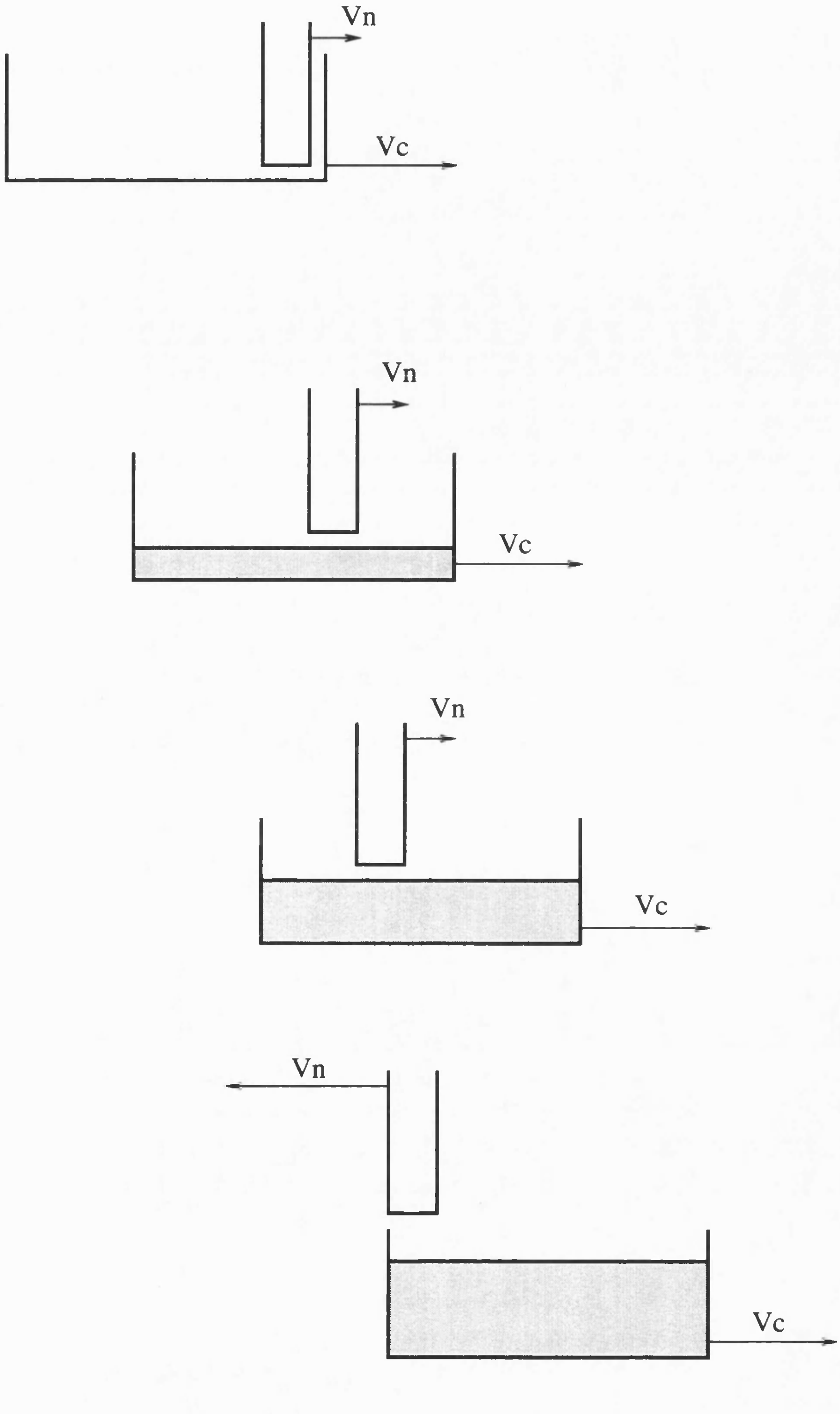


Figure 6.8 Optimised transition



Note: arrows represent horizontal velocity components

Figure 6.9 Relaxing the conveyor-tracking constraint

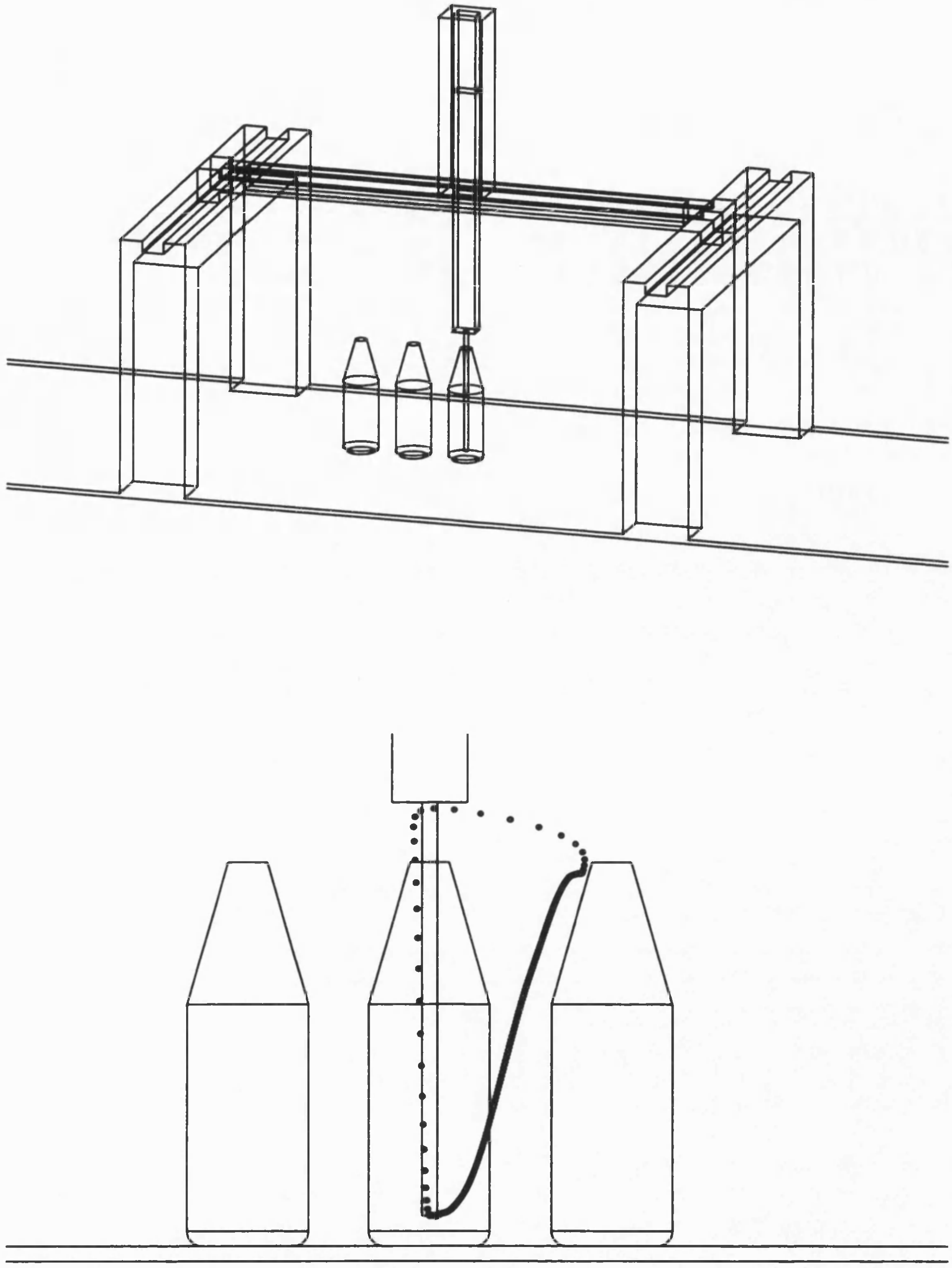
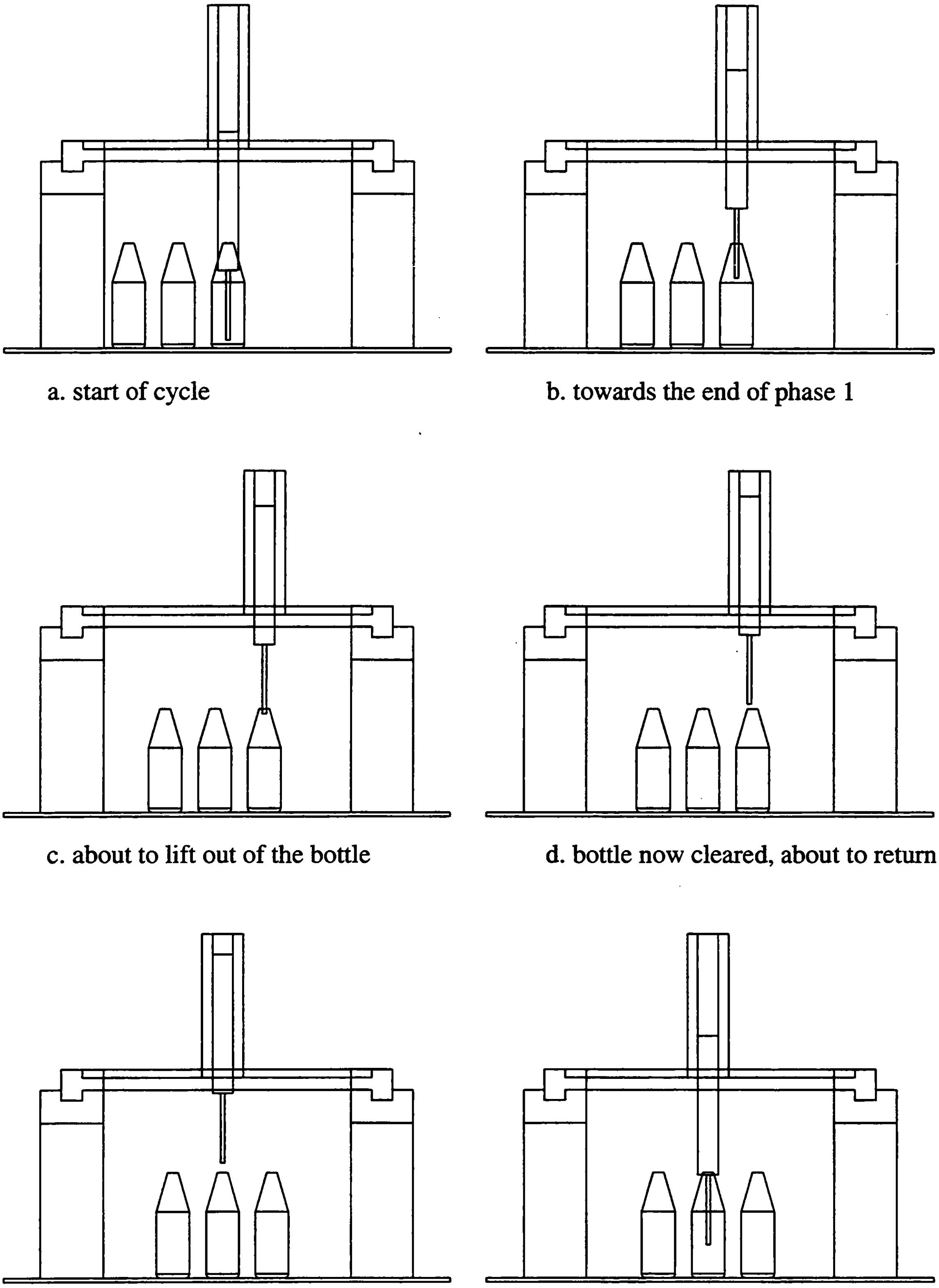


Figure 6.10 Screen shots from the computer animation



a. start of cycle

b. towards the end of phase 1

c. about to lift out of the bottle

d. bottle now cleared, about to return

e. returning

f. entering the next bottle

Figure 6.11 Screen shots from the computer animation

Chapter 7 Case study 2 - ice-cream cutter

This section describes how the methodology has been applied to the design of the ice-cream cutter introduced in chapter 3. It begins with further discussion of the requirements of the machine, and the specification initially given to the designers. It goes on to show that the requirements on their own do not give much information that is useful in the generation of design concepts. However, once a concept has been chosen, they can be used to assess the merits of different embodiments of the concept. In particular, it is shown how the methodology can be used to create a non-uniform crank motion for a linkage mechanism solution. This gives the designer performance data with which to evaluate the advantages and disadvantages of driving the linkage with a non-uniform motion as opposed to a constant speed.

7.1 Specification

This case study provides another example of the design of a continuous processing machine. Figure 7.1 illustrates the difference between the continuous and intermittent processes. Part a) of the figure shows the continuous process. Here, the extrusion of ice-cream through the nozzle continues, even while the bars are being cut and are falling onto the conveyor. The cutter path needs to slope downwards because of the need to track the extrusion as the cutter moves across the width of the nozzle. This contrasts with the intermittent process, shown in part b) of the figure. Here the extrusion stops after one bar's worth of ice-cream has appeared. The blade or cutting wire then cuts the bar, and returns to its initial position. Once the blade has moved out of the way, the next bar is extruded, and the process goes on. This has the advantage of being a more simple process to design. However, the limit on production rate is greater for the intermittent process, and this provides the motivation for designing a system that cuts a continuous extrusion of the product.

A brief specification was provided to indicate the basic requirements of the machine. This included a number of factors that directly relate to the motion of the cutter. There are tolerances on the flatness of the cut surfaces of the bars, and also on the speed of cutting. An additional challenge is presented by a desire to operate the machine over a range of cutting speeds and production rates. Numerical values for these are given below.

- nominal bar thickness of 25mm
- product flatness within 1mm
- cutting speed to be within 10% of the nominal operating value
- cutting speeds between 750mm/s, and 2250mm/s to be possible
- production rates of between 50 and 200 bars per minute to be achievable

Another requirement is the use of a standard heated wire to act as the cutter. This is seen as a more reliable cutting tool for this type of product than a sharp blade.

The design task can be summarised as being to determine whether these requirements can be realised with a physical machine, and if so, to determine the geometry of the machine.

7.2 Concept Selection

It is arguable that the items in the specification regarding the required motion do little to indicate what would be a suitable type of physical device to drive the cutter. They just require the cutting wire to move in a specified plane for a certain proportion of the machine cycle. This requirement does not determine which plane the cutting mechanism should lie in. The diagrams in figures 7.2 and 7.3 show ideas for mechanisms and motions which operate in either the vertical plane, or in the plane of the cut surface. Obviously it is not a problem to achieve a flat cut using the devices in figure 7.3. However, it seems they would be more suited to driving a rigid blade rather than a wire, and experience in the food industry suggests that the heated wire is the more reliable tool. This points to the conceptual motions shown in figure 7.2 as being more promising, as these can be realised by a planar mechanism guiding the heated wire. Of the two paths shown in the figure, the figure of eight would give the opportunity to cut two bars on each cycle. However it would be very difficult to manipulate the path in order to cater for the requirements of operating over the ranges of cutting speed and production rate. Obviously the slope of the cutter path must be greater for higher production rates, in order for the cutter to match the higher extrusion speeds. It would be simpler to orientate the non-intersecting path shown in part a) of the figure for operation at higher extrusion rates, and it is therefore considered that the open path is the most promising concept to investigate.

It is worth at this stage describing briefly how the mechanism concept can be made to work. In reality it would be necessary to have two identical mechanisms on opposite sides of the extrusion nozzle. The cutting wire would be supported at each end by the coupler points of the linkages, and it is therefore the motion of the coupler point that needs to be generated. Another practical issue is how the linkage is driven. The simplest method is to drive its crank at constant speed. However, in order to run the machine at the different operating conditions, a clutch is also necessary. This is because for any particular cutting speed there is a corresponding maximum production rate. This occurs when the input crank is driven continuously at constant speed. Lower production rates can be achieved for the same value of cutting speed by using the clutch to introduce a dwell into the cycle. The task of designing such a mechanism using the existing constraint modelling techniques is described in the next section.

7.3 Existing constraint modelling approach to mechanism design

A constraint modelling approach to the design of the ice-cream cutter was applied in earlier work (Kenney, Rentoul, Twyman, Kerr and Mullineux, 1997). Initially this involved selecting a mechanism to match a path that satisfies the spatial constraints, and then using optimisation to adjust link lengths, operating speed, and dwell duration in order to match target values of cutting speed and production rate. The mechanism selection was carried out using the techniques described in chapter 2, and a four-bar linkage was selected and optimised for one combination of cutting speed and production rate. The optimisation was carried out using error functions to account for constraints imposed by performance and clearance requirements.

This work was taken further by investigating how much of the specification (in terms of range of operating conditions) can be met by any particular linkage. It became apparent that linkage mechanisms have inherent limitations for this particular task. These were as follows.

- the speed of the coupler point is never constant over the whole cutting region, although it can be made to fall within the tolerances specified (see figure 7.4)
- the proportion of the cycle taken up in the cutting phase is limited, thus there is a limit on the maximum production rate achievable.

In fact the linkages found in the catalogues were only capable of operating at speeds of 75-115 rpm when performing at the lowest cutting speed. Clearly this falls well short of the desired maximum production rate of 200 products per minute. However, by selecting a mechanism with a relatively fast return phase, the constraint modeller has been able to produce an optimised linkage with a maximum speed of 142 products per minute. The work reported in the cited paper has subsequently been carried on and the extensions are discussed in the following sections.

7.4 Non-uniform motion

By considering the limitations described in the previous section, it follows that if a linkage is driven with a non-uniform motion, then the opportunity exists to obtain a more uniform cutting speed and possibly even higher production rates. The problem is how to create a suitable non-uniform motion profile for the crank. This section describes how this problem can be addressed by using the methodology introduced in chapter 5. It begins with a discussion of the construction of the parametric timing diagrams, and goes on to show how these can be used as an input to the mechanism selection techniques to create a linkage together with its crank motion diagram.

7.5 Construction of the parametric timing diagrams

The first step in creating the parametric timing diagrams is to split the motion up into its horizontal and vertical components, and consider the constraints on each of these. It is then possible to fit curves on the diagrams that match the constraints thus creating instances of the generic motion. These can be evaluated according to other constraints, such as for example, those regarding the quality of motion.

The orthogonal components for the ice-cream cutter are shown in figure 7.5. It is useful at this stage to consider the events that occur during the cycle. These are the instants during the cycle at which some notable action occurs, for example the start and end of a processing step. The cutting wire can be said to carry out two processes during its cycle: the first being the cutting action itself, and the second being the return to the start of the next cycle. From a point of view of the quantifying the constraints on the cutter motion, four events in this cycle are of interest, occurring at times *t1, t2, t3,* and *t4*. They take place during the two processes carried out by the cutter and are defined as follows.

- Event 1 - cutting begins
- Event 2 - cutting ends
- Event 3 - the cutting wire passes into the area underneath the nozzle
- Event 4 - the cutting wire clears the area underneath the nozzle

Thus, events 3 and 4 take place during the return phase of the motion, rather than at the start and end of the phase. In addition, *t5* is the cycle time of the motion.

Next the constraints on displacement, velocity, acceleration and jerk at these time points are considered. For convenience, *t1* is chosen to be the start of the cycle, and the corresponding *xy* position is specified in order to fix the location of the path in space. The origin of the *xy* space is chosen as lying on the nozzle centreline at the bottom of the nozzle (shown in the sketch in the lower part of figure 7.5). Hence *x1* equals half the nozzle width, and *y1* is chosen arbitrarily as some suitable clearance below the nozzle.

Having specified $t1$, $x1$, and $y1$, the values of $t2$, $x2$ and $y2$ follow automatically from the knowledge of cutting speed, production rate, nozzle width and product thickness. The cycle time, denoted as $t5$, is obviously obtained from the production rate as well.

By definition of events 3 and 4, the values of $x3$, and $x4$ are obviously the same as $x2$, and $x1$ respectively. However it is necessary to make arbitrary initial selections for the values of $t3$, $t4$, $y3$, and $y4$. These must also be made to lie within limits imposed by other constraints. An arbitrary selection must be made for two reasons.

- If no values are specified, it is not possible to fit curves through the other displacement constraints in x and y such that the curves combine to give an open path in space. Curve fitting techniques in this case give a straight line path.
- The values chosen must be considered arbitrary and free to be manipulated because there are no precise requirements at these instants. They are subject to inequality rather than equality constraints. It is just necessary for the values to lie within certain ranges to satisfy the clearance conditions. If they were to be fixed then the process of generating the motion profiles would be artificially over-constrained.

The important point to note here is that, having made certain arbitrary choices, the constraint modeller's optimisation procedure can be used to manipulate them. This can smooth out any unwanted characteristics they would otherwise impose on the path.

A suitable constraint on the value of $t3$ can be determined by considering its relationship with $t2$, the point at which a cut bar begins to fall onto the conveyor. It is necessary that the cutting wire, on its return to the start of the next cycle, does not touch this bar of ice-cream as it falls. The question is whether to allow the cutting wire just to miss the block of ice-cream, or whether to ensure that it is lying on the conveyor by the time the wire returns to position $(x3,y3)$. The latter has been chosen in this case study because, although it imposes an additional limit to the maximum production rate, it does give a maximum 'safety' margin. It is also very simple to determine how long it takes for a cut bar of ice-cream to fall onto the conveyor, because the height above the conveyor can be calculated easily.

In order to prevent the cutter's motion from fouling the advancing face of the ice-cream extrusion it is necessary to specify limits to the values of $y3$ and $y4$. The lower limit is conveniently determined from the height of the ice-cream blocks lying on the conveyor belt. In fact this constraint must be satisfied at all times between $t3$ and $t4$, but because of the nature of the curve fitting techniques, specifying the constraint at just the start and end of this zone is sufficient. The upper limits on $y3$ and $y4$ can be determined by one of two methods. The first allows the wire just to miss the extrusion, and therefore allows the shortest possible path to be created. The second allows a further margin of safety to be introduced by limiting $y3$ and $y4$ to the position of the extruded face at the instant that the next cycle's bar is about to fall. These limits are shown in figure 7.5.

In summary then, the time constraints specified in order to construct the timing diagrams are,

$$t1 = 0$$

$$t5 > t4 > t3 > t2 > t1$$

and for convenience,

$t3$ - $t2 \geq$ time required for the ice-cream block to fall onto the conveyor

The form of the constraints on the displacement diagrams suggests that the most suitable geometric entities to construct particular instances of the motion profiles are a line segment for the cutting process, and a free-form open curve for the return phase of the motion.

The use of the line segment is obviously trivial, but the methods used to fit a curve between points *t2* and *t5* need further explanation. Constraints must be imposed on the curve fitting problem and this involves specifying precision points on any of the derivatives of the motion which the displacement curve and its derivatives must pass through exactly. It is necessary to provide suitable boundary conditions so that the curve starts and finishes at the correct place. These consist of the displacement, velocity, acceleration and jerk values at times *t2* and *t5*. By specifying constraints on the first three derivatives of the displacement, it is ensured that the overall motion profile is smooth and continuous in displacement, velocity and acceleration. This helps partially to satisfy the constraints on quality of motion described in chapter 3. Along with the boundary conditions, the open curve also has to pass through the intermediate points on the displacement diagrams at times *t3* and *t4*. This means that a total of ten precision point constraints need to be imposed on each of the orthogonal components of the motion in order to create smooth and continuous displacement functions.

Once the curves are fitted to the x and y diagrams the resulting functions are evaluated at a number of equal time intervals. This forms an array of xy coordinates that can be plotted in the xy plane, as illustrated in figure 7.6. At this stage it is necessary to assess whether the resulting motion profile is a suitable one, or whether it is even in some sense optimal. This means checking that the other constraints in the design task are satisfied. If they are not, which is likely to be the case given that certain arbitrary choices have been made, then the motion can be recreated with a new set of parameter values. This can be done manually by the user, or automatically by the optimisation algorithm in the constraint modeller. This is described in the next section.

7.6 Optimisation of the timing diagrams

A basic configuration for the machine system is defined by the following variables.

- nozzle width
- product thickness
- cutting speed
- production rate
- height of the nozzle above the conveyor
- clearance between the cutter path and the nozzle

Having specified values for these parameters, the motion profile for the cutter can be optimised by choosing suitable design variables, constraints, objectives, weightings, and initial values. The rules or objectives for this work are to create a motion profile that is a best compromise between minimum cycle time, and minimum peak velocity and acceleration. For this reason, the production rate is chosen to be a design variable. It is also necessary to specify the arbitrary variables *t3, t4, y3,* and *y4* as being free to be altered by the optimisation algorithm. The constraints imposed are those described in the previous section.

It is useful to carry out the optimisation a number of times in order to investigate the effect of changing the starting values for the design variables, and the weightings for the rules. This is because of the tendency of the search techniques employed in the current implementation of the constraint modeller to find locally optimal solutions. Some interesting optimisation results are shown in figures 7.7-7.9. They were obtained by running the optimisation algorithm under different sets of starting conditions. The top part of each figure shows the resulting orthogonal components of the motion. The lower parts show the paths in *xy* space in relation to the upper and lower bounds imposed on values *y3* and *y4*.

An interesting result (figures 7.7, 7.8) is that the maximum production rate can be increased to 178 bars per minute (bpm) which, although still below the initial specification for the machine, compares favourably with the 142 bpm achieved with the constant speed mechanism. By altering the clearance constraint to allow the cutting wire just to miss the extrusion on the return stroke, this can be increased to 228 bpm (figure 7.9). As the timing points have now been specified, there is no guarantee that a uniformly driven mechanism will be appropriate. The creation of a non-uniform crank motion is discussed in the next section.

Summary of constraints

X displacement and velocity

- achieve desired cutting speed
- locate cutting phase under nozzle

Y displacement and velocity

- match extrusion rate
- locate cutting phase under nozzle
- arbitrary points on the return used to generate valid return motion, bound by collision-avoidance constraints.

Acceleration and jerk

- smooth transition between cutting and return phases achieved with zero acceleration and jerk at t1 and t2.

Optimisation

- minimise cycle time, peak velocity and peak acceleration, i.e. investigate compromise between high production rate and reasonable velocity and acceleration levels.
- variables
    - production rate
    - time and y-displacement of arbitrary points on the return

7.7 Creation of the non-uniform crank motion

Having obtained an optimised path the following procedure is used to create a linkage with a non-uniform crank motion profile that traces the path as closely as possible.

1. Select a mechanism that matches the shape of the path well.
2. Optimise the mechanism geometry so that the shape match is as close as possible.
3. Use the constraint modeller to try to position the coupler point as closely as possible to each timing point on the optimised path.
4. At each of these positions record the crank angle.

The first two steps involve the use of the same selection and optimisation techniques used in previous constraint modelling studies. The important difference is that the path being matched has been designed such that the timing around the path is considered optimum. This would seem to suggest the optimisation should be performed using the time dependent Fourier coefficients as objectives to match. However, experience has shown that optimisation on the basis of these leads to a significant compromise on the shape of the resulting path. What is required here is an extremely close match to the target path: timing resulting from the constant speed input is of little interest because the speed is to be controlled by the non-uniform motion. Having obtained a mechanism that closely matches the shape of the designed path it is straightforward for the constraint modeller to determine the crank motion profile required to match its timing. This is achieved by running an optimisation process to position the coupler point of the mechanism as closely as possible to each timing point on the path. The variable manipulated in the optimisation is the crank angle, and this is recorded when the coupler is as close to a point on the path as it can get. A schematic diagram is shown in figure 7.10.

Clearly errors in both position and timing are inevitable with this design technique. However, its use is justified by the fact that the constraint modeller can employ the optimisation algorithm to ensure that the results lie within tolerance. The chance of success with the technique is increased by taking an 'optimised' path that appears at the outset to be suitable for a particular mechanism type.

An interesting result from this approach is that the number of potential mechanism solutions to the problem is greatly increased. The reason for this is as follows. When designing the mechanism to be run at constant speed, the linkages that exhibited a 'slow' return phase are discarded even if they can trace a very straight path over the cutting region. This is because the 'slow return characteristic places a greater limit on maximum production rate than the 'fast' return mechanisms. However, when the timing, and hence cutting speed, of the mechanism is to be controlled by a servo motor, any linkage with a suitably flat path over the cutting region is a potential candidate solution to the problem; even the 'slow' return mechanisms. This gives rise to another set of design trade-offs. Mechanisms with a 'slow' return under constant speed input may require larger changes to the crank speed, and hence experience greater levels of angular acceleration when tracing the path. This has an effect on power consumption. The 'fast' return linkages, on the other hand, may not require much adjustment to their timing, but it must still be possible for the control system to trace the non-uniform profile.

The graphs in figures 7.11 and 7.12 show crank angle and transmission angle for two mechanisms that trace the second of the optimised paths obtained in the previous section.

The first is a linkage that, under a constant speed input, has a 'slow' return characteristic. The second is a 'fast' return.

7.8 Alternative approach to non-uniform motion

An alternative approach to the creation of the crank's non-uniform motion profile uses a different form of parametric timing diagram. It involves starting with a mechanism that traces a suitable path, rather than the required motion. Then the timing is adjusted to suit the motion constraints of the problem.

7.9 Concluding remarks

This case study has demonstrated a number of important points regarding the use of constraint-based parametric timing diagrams in motion design. It is an example of a machine design problem where type synthesis is rather involved, and detailed work on motion design begins only when specific design concepts are being investigated. Most importantly, it is an example of a design problem where the specification cannot be fully met by even the most promising design concept, namely the four-bar linkage. The constraint-based approach to the motion design has however provided an environment in which the discrepancy between actual range of performance and specified performance can be quantified and minimised.

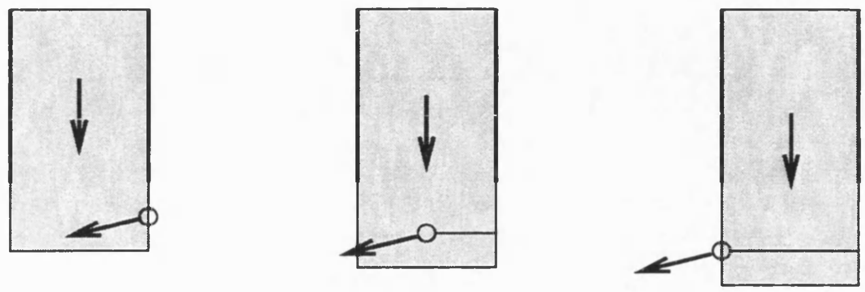
The constraint-based timing diagrams have been successfully used in conjunction with standard mechanism selection techniques to generate non-uniform crank profiles for linkages. This results in a machine that can meet much of the original specification.

Constraints of the types described in chapter 3 have been identified and relevant ones included in order to construct the timing diagrams. These are listed below.

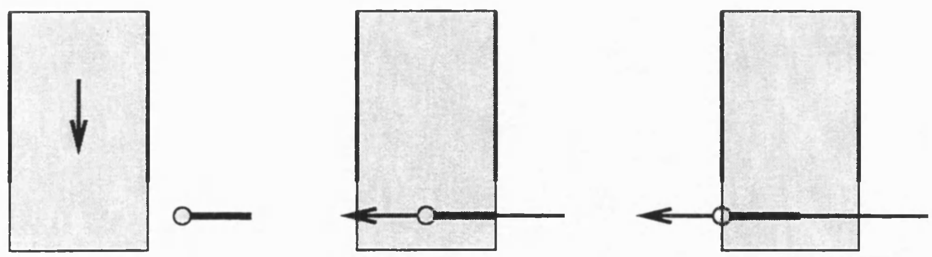
- quality of motion constraints, which apply to the curve fitting.
- clearance constraints, which impose bounds on acceptable values in both space and time.
- performance constraints, which impose a region of precision points, i.e. a straight line path.
- commercial constraints, which influence the maximum production rate.

It has been necessary to insert some artificial 'precision point' constraints on the timing diagrams in order to create a two-dimensional path. However, the use of optimisation has minimised the detrimental effects of over-constraining the design task in this way. Having set up the basic diagrams a line segment and open B-spline curve have been used to interpolate a profile through the constraints. The open curve has boundary constraints on displacement and its first three derivatives, and also has intermediate precision points on the displacement.

It has been possible to minimise the period of the motion despite the fact that the cycle contains processing events of fixed duration.



a. Continuous processing
Cutter moves diagonally downwards as the extrusion continues



b. Intermittent processing
Cutter moves horizontally through stationary extrusion

Figure 7.1 Continuous and intermittent extrusion

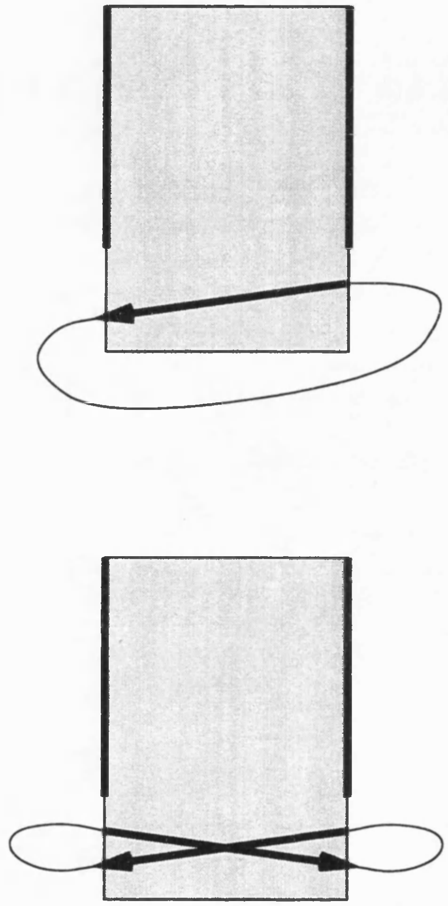


Figure 7.2 Closed cutter paths in a vertical plane

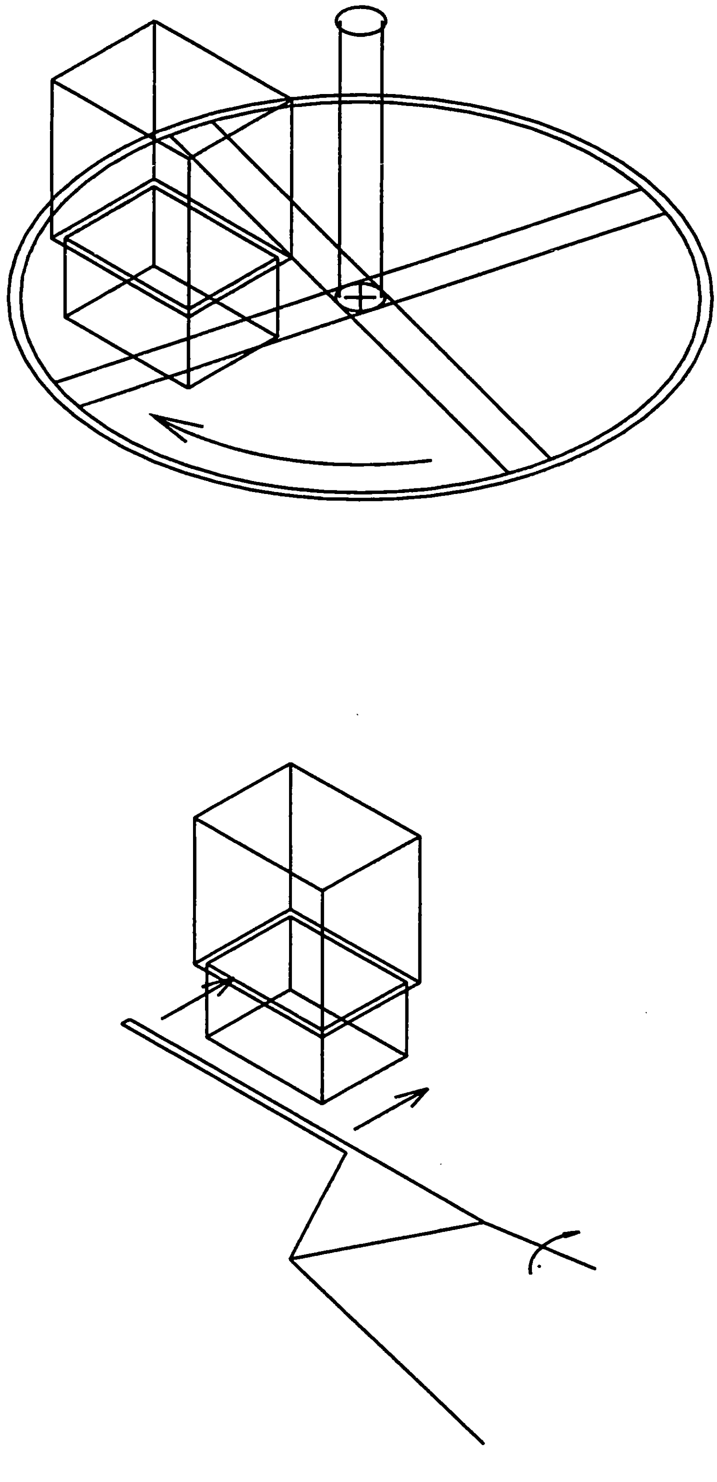


Figure 7.3 Cutter motion in the plane of the cut surface

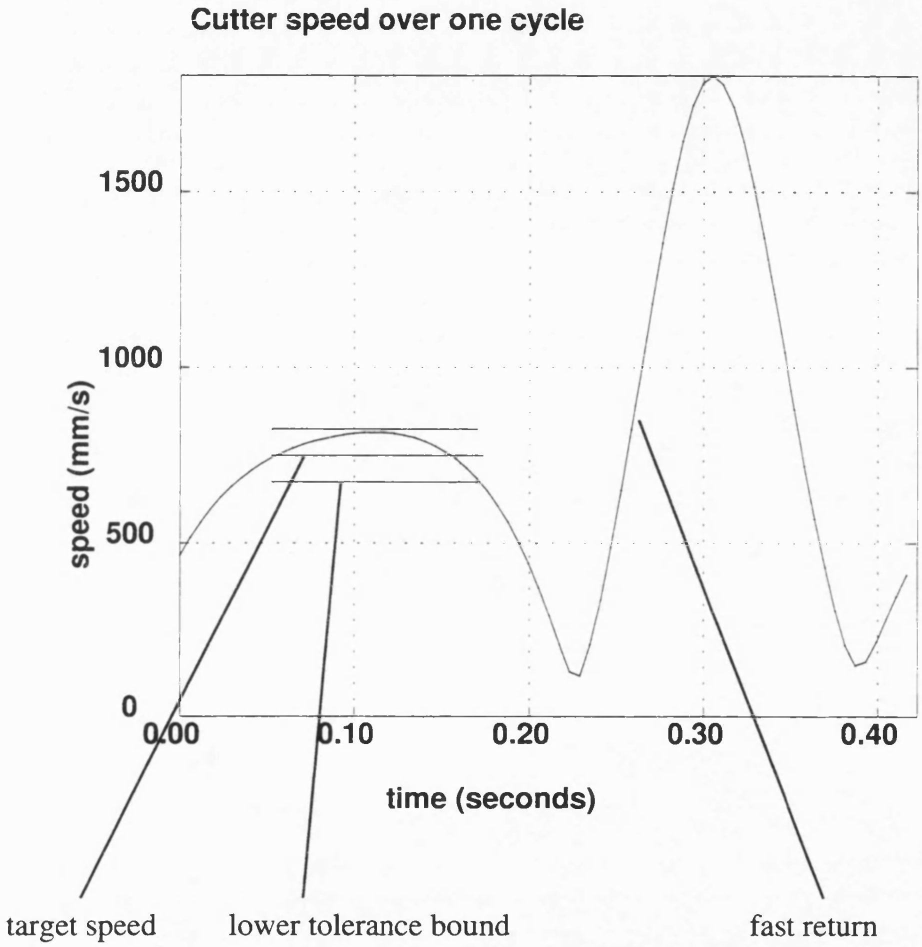
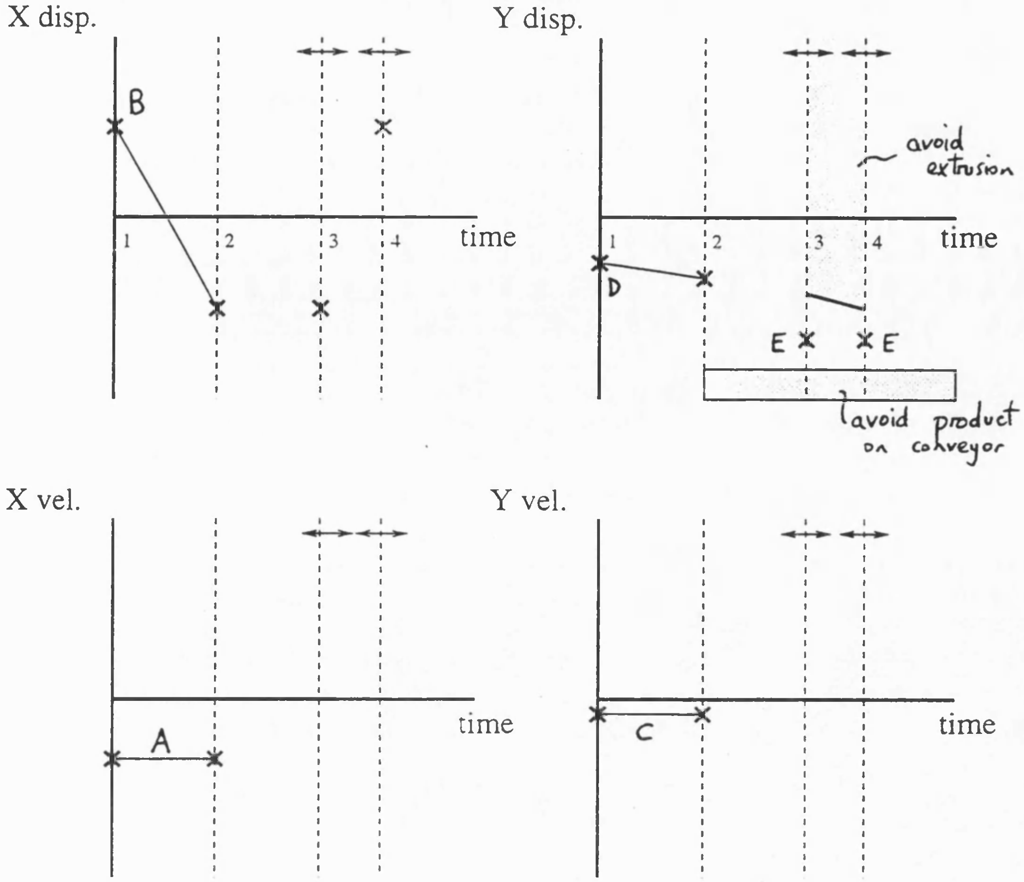
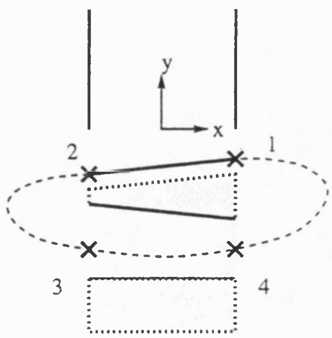


Figure 7.4 Cutting speed of optimised, uniformly driven mechanism





Summary of constraints

X displacement and velocity
A achieve desired cutting speed
B locate cutting phase under nozzle

Y displacement and velocity
C match extrusion rate
D locate cutting phase under nozzle
E arbitrary points on the return used to generate valid return motion, bound by collision-avoidance constraints.

Acceleration and jerk
smooth transition between cutting and return phases achieved with zero acceleration and jerk at t1 and t2.

Optimisation
Minimise cycle time, peak velocity and peak acceleration, i.e. investigate compromise between high production rate and reasonable velocity and acceleration levels.

- variables
    - production rate
    - time and y-displacement of arbitrary points on the return (E)

Figure 7.5 Parametric timing diagrams

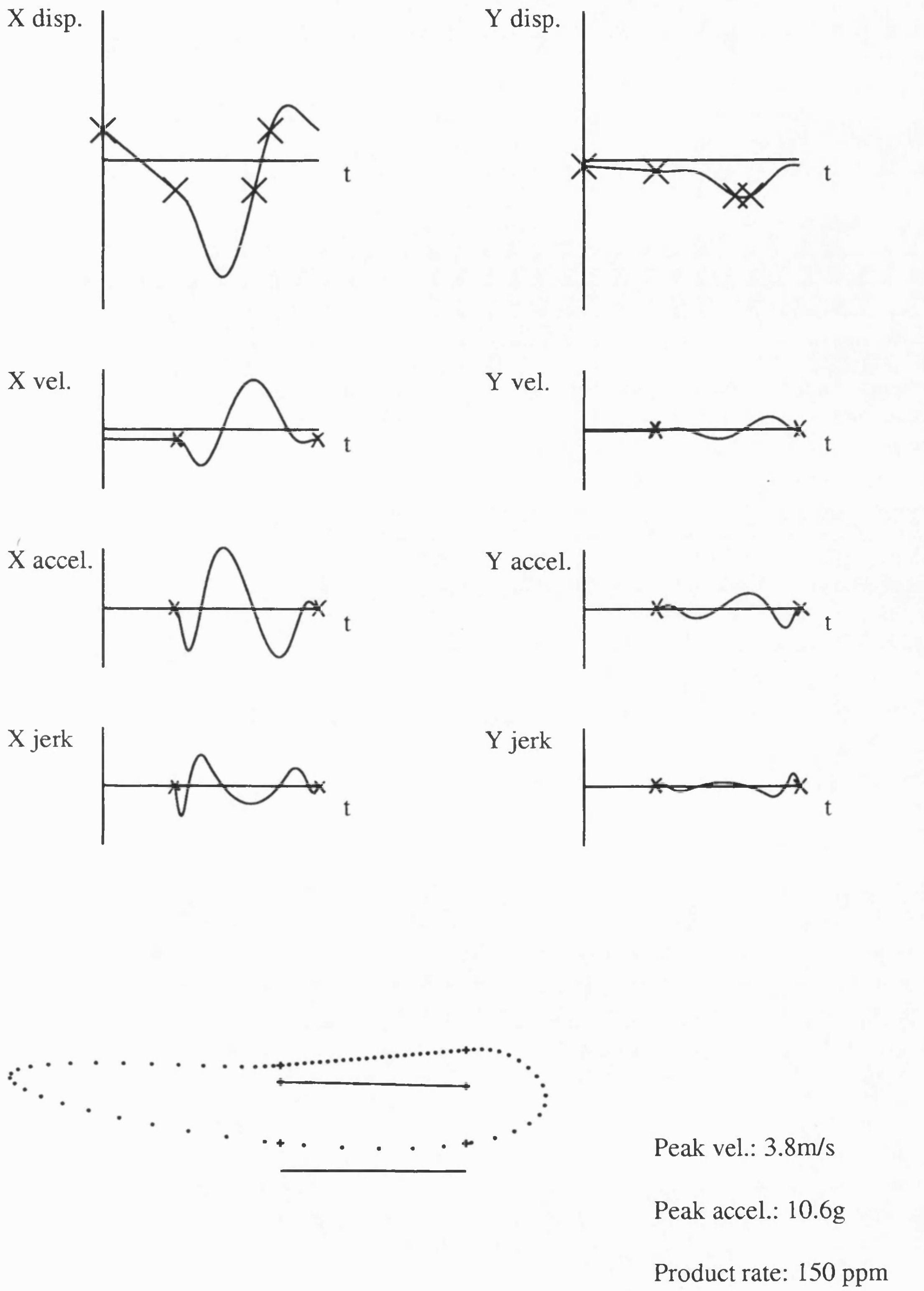
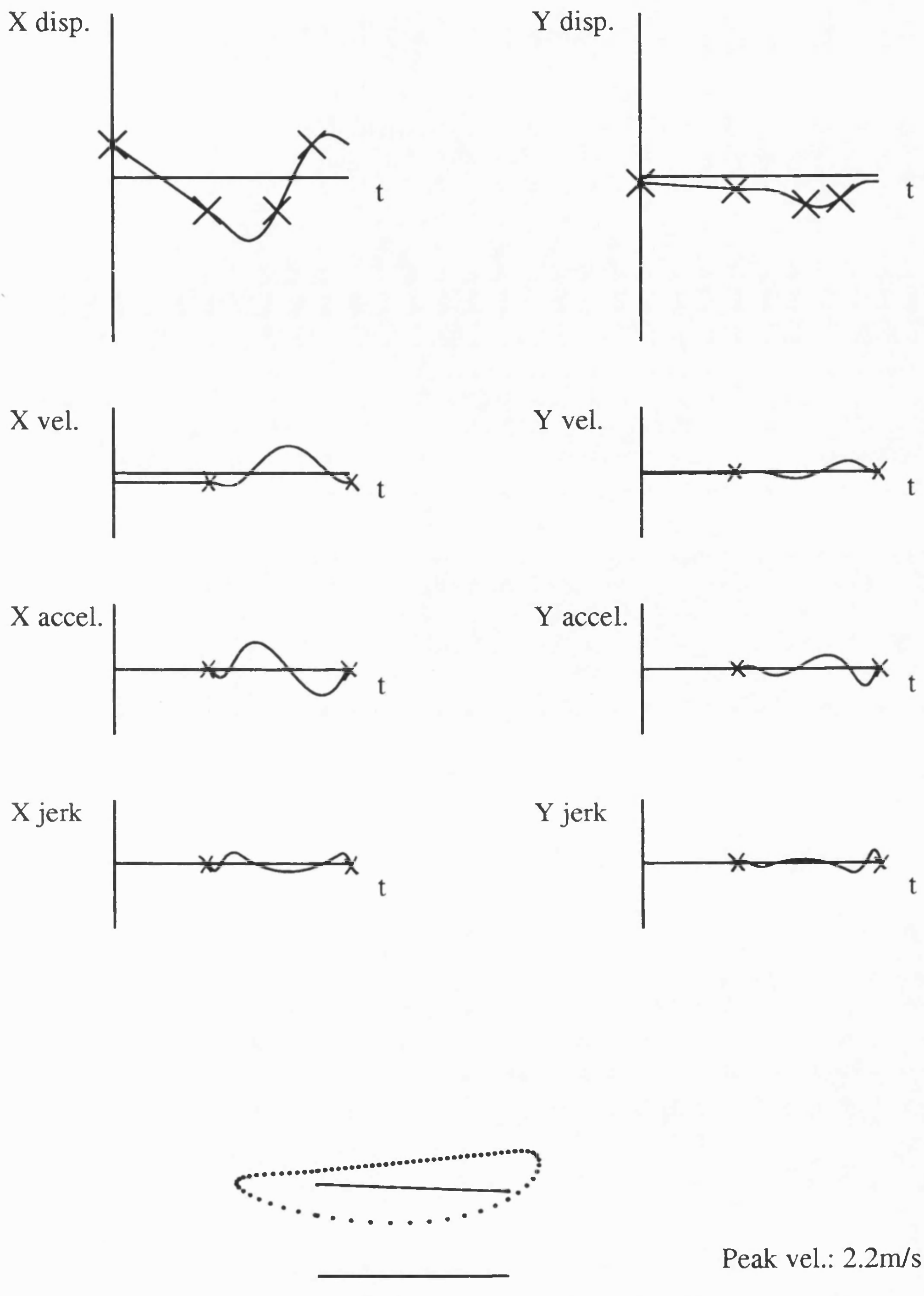


Figure 7.6 Profile generated from initial values for the arbitrary precision points



Peak vel.: 2.2m/s

Peak accel.: 6.0g

Product rate: 178 ppm

Figure 7.7 Optimised results - case 1

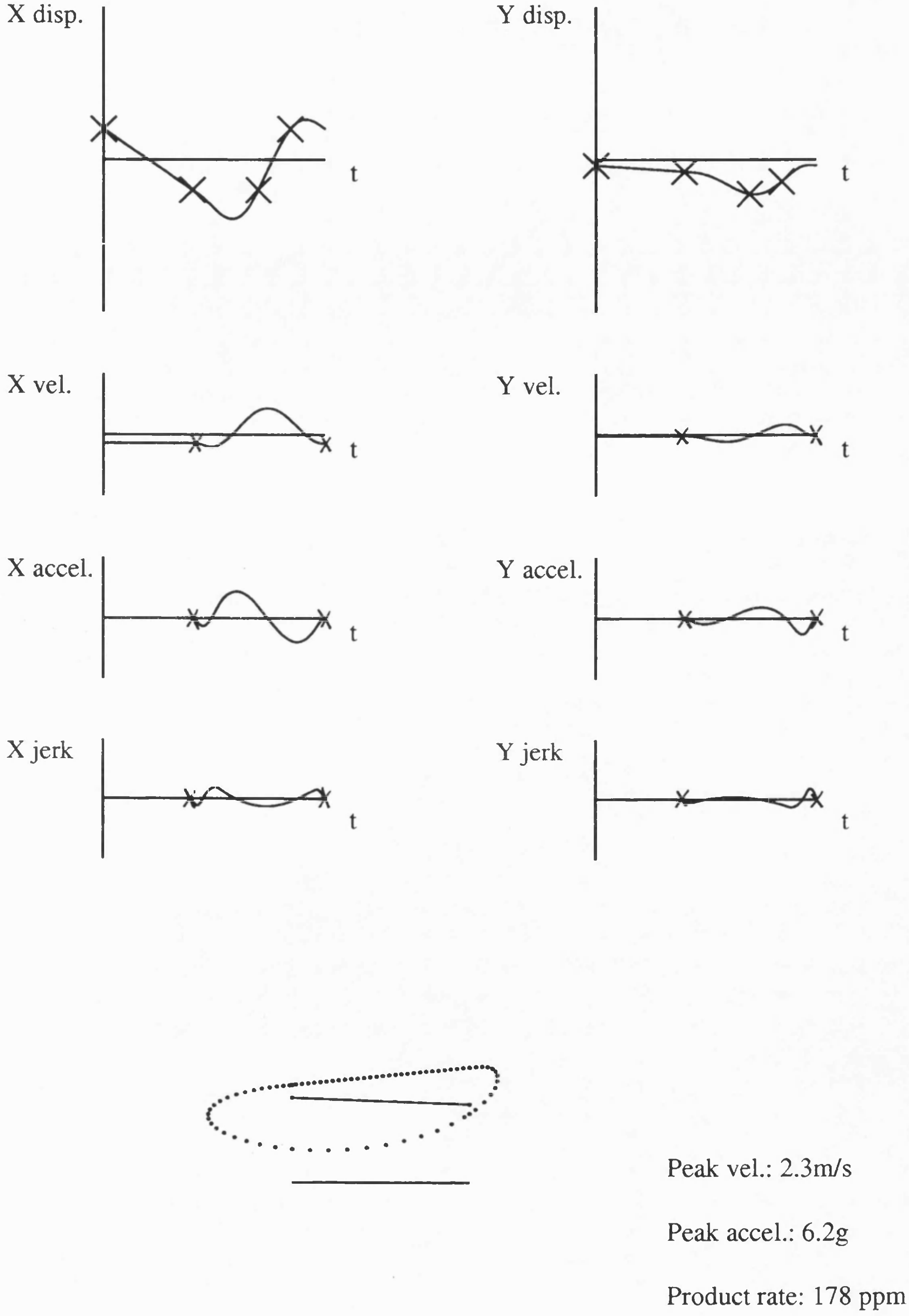
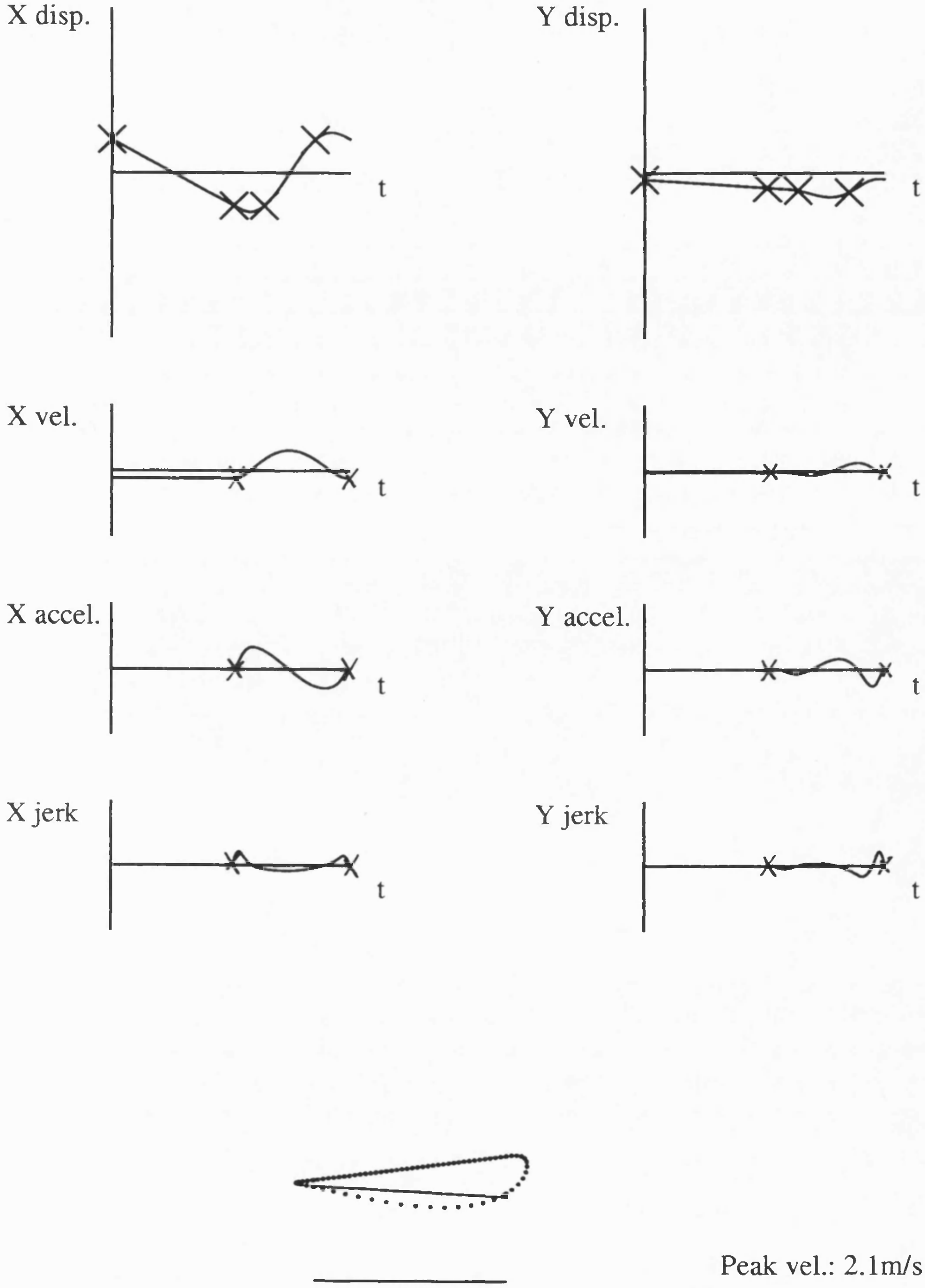


Figure 7.8 Optimised results - case 2



Peak vel.: 2.1m/s

Peak accel.: 7.9g

Product rate: 228 ppm

Figure 7.9 Optimised results - case 3

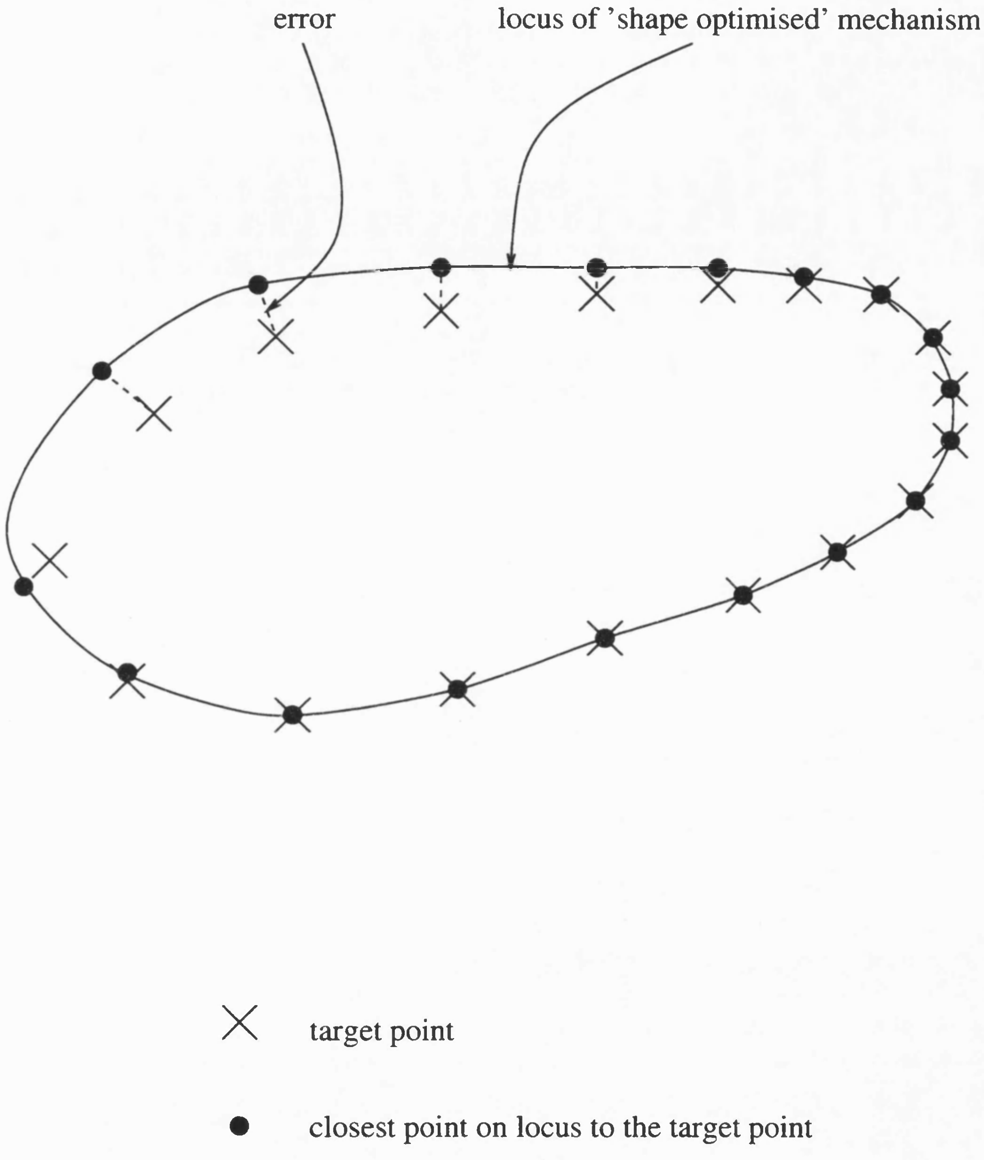


Figure 7.10 Path error from non-uniform drive design technique - exaggerated for illustrative purposes

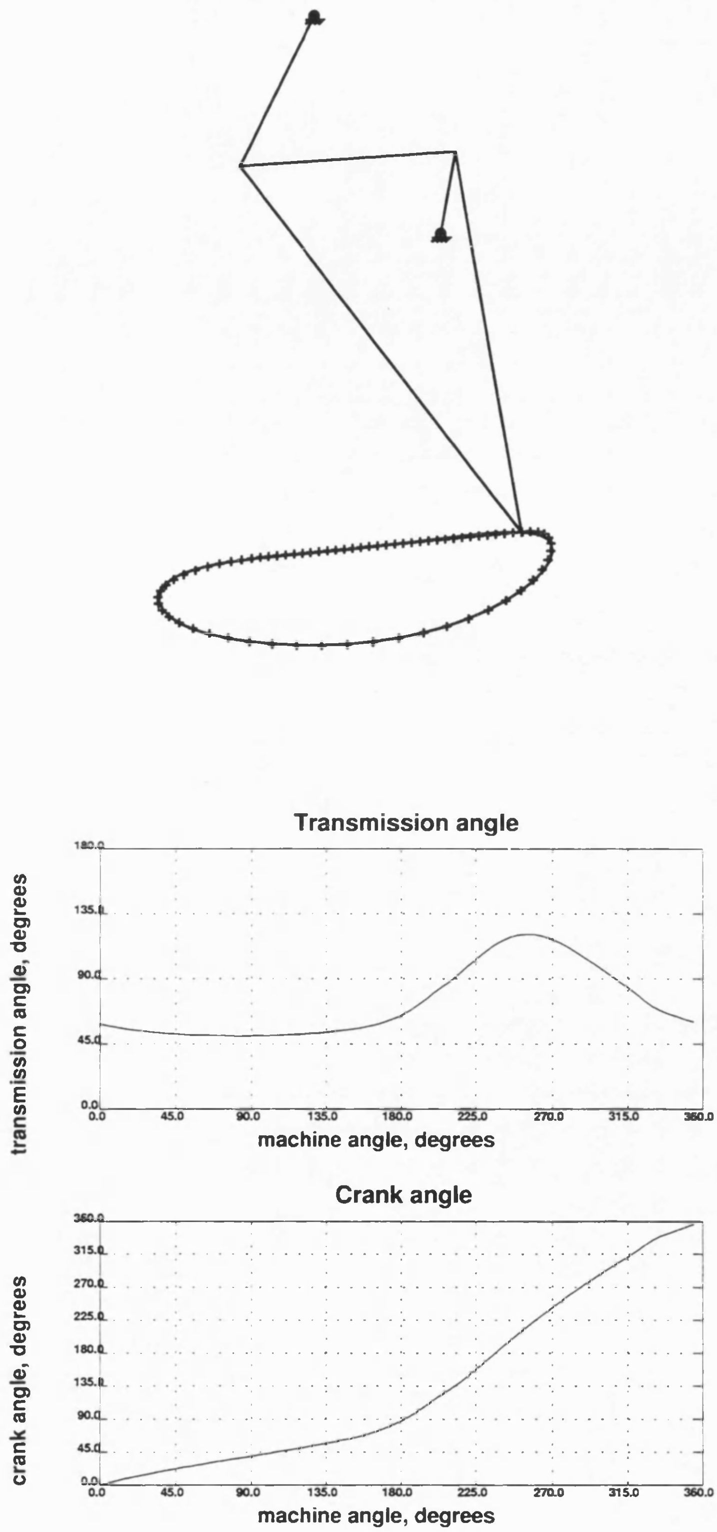


Figure 7.11 Four-bar mechanism for optimised path case 2

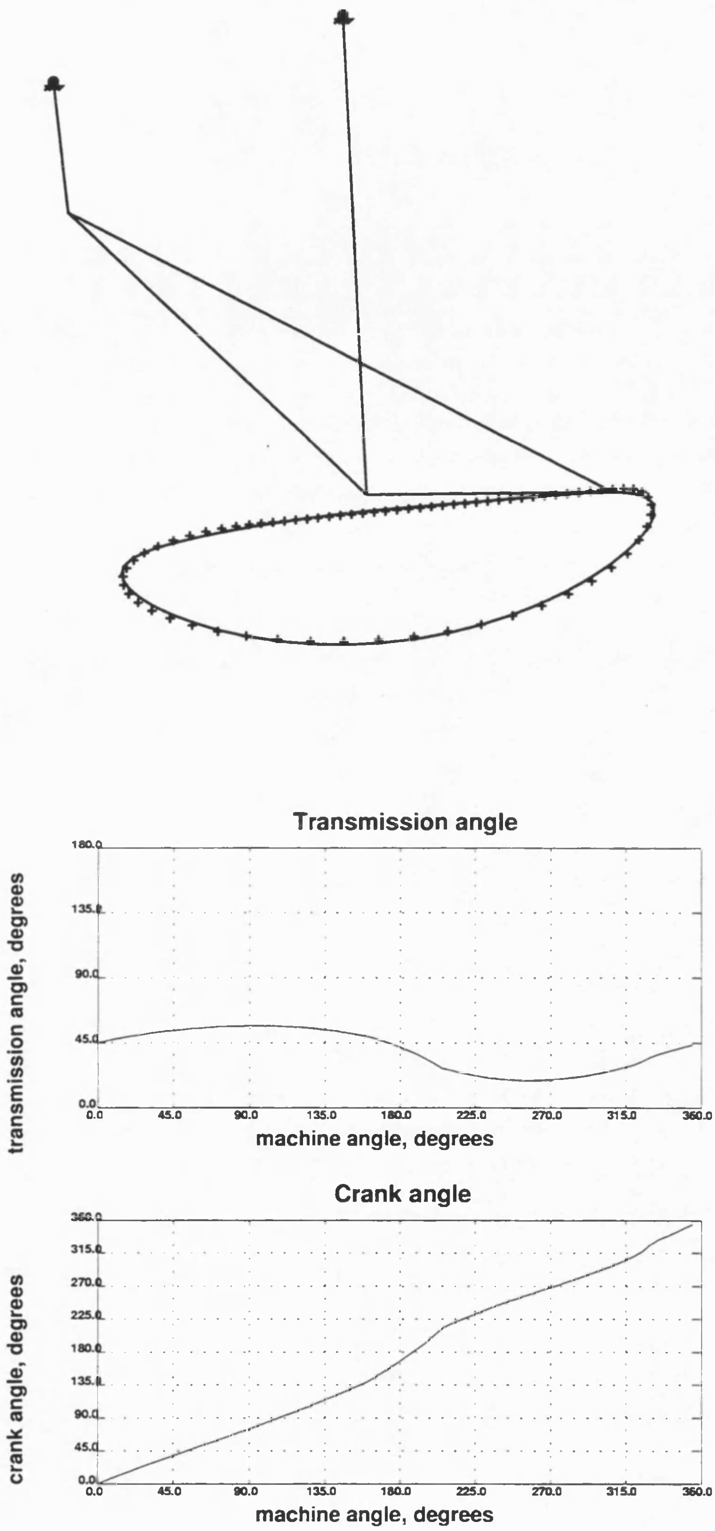


Figure 7.12 Alternative four-bar mechanism for optimised path case 2

Chapter 8 Case study 3 - can handling machine

This chapter describes how the approach to motion design has been used to tackle the problem of designing a mechanism to load cans into a forming machine. A geometrical layout of the forming machine already exists, so the problem presented here is one of creating additional hardware to work in conjunction with it.

8.1 Concept Selection

A schematic of the machine introduced in chapter 3 is shown in plan view in figure 3.3. It consists of a large turret that contains 36 sets of tooling to process blank, unformed cans. Processing occurs on a continuous basis while the turret revolves at constant speed. When a blank can is loaded into an empty station (or pocket) on the turret, it is grasped by part of the tooling and, as the turret revolves, more tools move into position to carry out work on it. There is one set of tools for every station, and this revolves with the station, so the turret never has to be brought to rest to enable the forming operations to be carried out. This mode of operation allows higher production rates than one that uses an intermittent turret motion. However, the machine does require a dedicated device to load the blank cans into it, and to remove finished cans after the forming operation has been completed. The loading procedure cannot occur instantaneously though, because a tool on the main turret is required to grasp the can from the bottom, and a non-zero time is needed for it to do this. The transfer is therefore rather like a baton change in a relay race: one device has to 'hand' a can to the other, without the can experiencing free flight. Hence, to load cans reliably into the machine, the loading device must track the position of a pocket on the main turret for sufficient time for the main turret's tooling to take hold of the can. This means that, over a certain portion of the turret's circumference, it must hold a can in a fixed position relative to the tooling to enable the transfer to take place. The dashed line in the figure represents the path of a can through the system. The thicker lines show the regions where the loading and unloading operations take place. For simplicity, the following discussion deals only with the design of the loading mechanism, as the unloading operation is basically the same operation in reverse.

A number of design concepts can be drawn up to tackle the problem of loading cans into the turret. Three potential arrangements are shown in figure 8.1. The first of these (figure 8.1a) consists of a chain that is driven around a bean-shaped 'cam' profile. Each link of the chain carries a pocket to hold an unformed can, and also a number of followers to ensure it remains in contact with the cam. The chain's path follows the circumference of the main turret pockets for a sufficient length to enable the transfer to take place.

A second concept (figure 8.1b) involves the use of pairs of connected levers, mounted on a second rotating turret. Each pair has a pocket on the end of the outer lever, and this is used to take an unformed can from a supply wheel and, as the whole assembly rotates, to carry it round to the main turret. The levers pass over a pair of stationary cams, and this causes the pocket's path to be diverted in such a way that it follows the circumference of the main turret for a short distance, thus allowing the transfer to take place. Two cams are required in the design to give sufficient freedom for the levers to match both the position and speed of the pockets on the main turret.

The third concept, shown in part c) of figure 8.1, is again based on a second rotating transfer turret. This carries pairs of actuators: one linear, one rotary. Each pair is arranged so that the can-holding pocket is mounted on the linear actuator, and this assembly is mounted on the rotary actuator. Over the 'transfer region' the pocket is forced to follow the required path by adjusting its angle and distance from the turret centre by suitable control of the actuators. In effect the actuators are only required to superimpose small adjustments on the rotary motion of the turret, so that the pockets are diverted to track the main turret's circumference during the transfer operation.

One problem common to all three of these concepts is to decide how many pockets to use on the transfer device. It is possible to use fewer than the 36 pockets on the main turret by arranging for each one to serve a set of the main turret pockets. This has many of the advantages associated with the use of smaller numbers of components: manufacturing costs are reduced, and there are improvements in reliability, and ease of manufacture, assembly, and maintenance. Such a transfer system would have to operate at a higher speed than the main turret however, and it is not simple to determine whether this would have a significant effect on the design.

The following sections describe how motion profiles for the last two of these design concepts can be designed by use of constraint-based parametric timing diagrams. They also show how design optimisation can be performed to obtain a compromise between good quality output motion, and good quality mechanism geometry. This in turn assists in making design decisions such as how many pockets to use on the transfer turret.

8.2 Construction of the parametric timing diagrams

Before sketching axes for the timing diagrams, it pays in this case to develop the sketch of the basic form of the motion, so that the phases and events are identified. This helps in the selection of a coordinate frame, which is obviously a prerequisite for creating the diagrams. The top part of figure 8.2 shows a transfer turret positioned in such a way that it overlaps the radius of the pockets on the main turret. The points 1, 2, 3, and 4 represent important events in the cycle of the transfer turret. These are as follows.

- Event 1: the transfer mechanism's path begins to follow the main turret path so that the transfer operation can take place.
- Event 2: the path breaks away from the main turret circumference after the can has successfully been grasped by the main turret's tooling.
- Event 3: the empty pocket on the transfer turret enters a region where it picks up its next blank can.
- Event 4: by this point in the cycle, the pocket has picked up a blank can.

It is assumed in this piece of work that the pockets on the transfer turret can pick up a can instantaneously, anywhere between points 3 and 4. This makes it convenient to assume a circular path between points 3 and 4. Furthermore, because this represents such a large proportion of the transfer pocket's cycle, it is also convenient to take the centre of the transfer turret as the origin of the coordinate system for the timing diagrams. An alternative would be to take the centre of the main turret, as this already exists.

Another point worth considering at this stage is that this motion design problem is essentially one of blending two circular arcs together in a smooth way. A question that needs to be answered is what form the motion should take during the transition from events 2 to 3 and from events 4 to 1. Two possibilities are shown in figure 8.2. The first of these shows a path that travels outside the transfer pocket radius during the transition, and the second shows a path that cuts inside to perform the blend. In order to evaluate the merits of these path shapes, the timing diagrams are used to produce example profiles of each one. These are then used as input data to computer models of the mechanisms to determine whether acceptable mechanism geometry can be created to trace them.

The sketched forms of the timing diagrams are shown in figure 8.3. These show the displacements in x and y components of the transfer pocket, relative to the transfer turret centre. Between times $t1$ and $t2$, the path follows the circumference of the main turret, and between times $t3$ and $t4$ it follows the transfer turret circumference. Because these portions of the cycle involve constant speed rotation, the paths in each direction are parts of a sinusoid. Clearly the radius and rotational speed of each turret, together with the coordinates of the turret centres must be known in order to determine the exact geometry of the path during these phases. The dashed lines in the diagrams represent the transition periods, which, in this case, are to be interpolated by open B-spline curves bounded by the displacement, velocity, acceleration, and jerk values of the sinusoids at times $t1$, $t2$, $t3$, and $t4$. For the purposes of satisfying the timing requirements of the transfer operation, $t1$ and $t2$ are best considered 'fixed', or at least tightly controlled. There is more scope to vary $t3$ and $t4$, because the pick up operation is assumed instantaneous. The approach taken is to encode these segments in the computerised versions of the timing diagrams in such a way that the path shapes illustrated in figure 8.2 can be generated by changing $t3$ and $t4$, and perhaps with small changes to $t1$ and $t2$ as well.

As indicated above, the values needed to define the sinusoidal segments on the timing diagrams are derived from design parameters such as rotational speed, turret radius, and the distance between turret centres. The following variables for the main turret are known.

- radius of the main turret.

- number of stations (pockets).
- rotational speed.
- approximate time required for transfer.

Of these, the first three are fixed by the existing design of the main turret, which in turn is influenced by the overall production volume required of the machine. The time required for the transfer operation is not exactly known. However, the experience of the engineers working on the design in the company concerned suggests that the minimum angle of overlap between the two turrets should be no less than 20 degrees of the main turret. Obviously a larger angle of overlap would allow more transfer time, but a limit comes from the overall 'packaging' of the machine. It is desirable for the loading and unloading turrets to occupy as small a portion of the main turret circumference as possible. This is because of the need to use the majority of the main turret's cycle time in actually carrying out the processing operations on the cans.

It is simple to determine the rotational speed of the transfer turret: it is just geared to the main turret in the ratio of the number of pockets on each turret. This is because there is a need for one pocket on the transfer turret to serve a fixed set of pockets on the main turret.

There is no hard and fast rule regarding the choice of radius for the transfer turret however. The packaging constraints described above indicate it should be as small as possible, though still allowing the 20 degrees overlap angle. There is also another factor involved however, and this comes from the fact that the two design concepts (the cam and lever system, and the actuator pair system) can only make small adjustments to the pocket's path in the transfer region (between *t1* and *t2*). More specifically, the cam and lever design can only make very small changes to the tangential speed of a pocket, although it can make reasonably large changes to the radial position. It is therefore sensible to design the transfer turret radius so that its tangential speed is approximately equal to the tangential speed of the main turret. The diagram in figure 8.4 shows how this has been done. For a given overlap angle and rotational speed, the transfer turret radius is set so that the speed at a radius determined by the intersecting chord equals the main turret's tangential speed. The distance between turret centres follows on directly from this choice.

An example motion profile produced with the diagrams for a transfer turret incorporating 12 pockets, is shown in figure 8.5. The sinusoidal segments in the timing diagrams have actually been represented by a series of points in the constraint modeller macro that created them. This is just to allow for simple visualisation, because the more important part is the overall path in the *xy* plane, produced by combining points from the *x* and *y* timing diagrams at a series of time values. This is more important as the points on the path are used directly by the constraint modeller as input to the kinematic model of the mechanisms. The values of each of the first three derivatives at the ends of the sinusoidal segments have been used as precision points for fitting the open curves to form the blends from one arc to the other.

The constraints treated in the discussion above fall into the categories of 'performance' and 'quality of motion' as described in chapter 3. However, there are also clearance conditions that need to be met by the design of the loading mechanism. These are not so much an issue for the design of the pocket motion, but are extremely important in determining the acceptability of the set of cams required in the second design concept.

The amount of space available for the loading mechanism to occupy is determined by examining a side elevation of the existing design of the main turret. Up to this point in the discussion, the radius of the locus of pockets on the main turret has, for simplicity, been referred to as the main turret radius. There are however, other parts of the turret which have different radii. By mounting the cams in a plane below the pocket locus, it is possible to 'undercut' the locus by a small amount, without causing a collision with the hard surfaces of the turret itself. For simplicity, a two dimensional model of the turrets, cams, and levers is used to determine suitable mechanism geometry. It is therefore sufficient to test for clash by determining whether any points on the pitch curves pass inside this smaller radius that lies beneath the pocket locus.

8.3 Synthesis and optimisation of the cam and lever geometry

This section describes how motion profiles of the type displayed in figure 8.5 have been used to help create suitable geometry for the second design concept, the cam and lever system. The motion profiles form the input to an inverse kinematic model of the design, which determines the pitch curves of the corresponding cams. This model is built by a macro program interpreted by the constraint modeller. Being a parametric model, it also allows the designer to investigate the effect of varying the number of lever pairs (and hence pockets) mounted on the turret. For simplicity and speed of calculation, just one pair of levers is treated initially. The rest are added after the geometry has been established, so as to check the feasibility of their spatial layout.

The model consists of three model spaces: one for each of the levers, and another for the transfer turret. It is illustrated in figure 8.6. Two lines are used to represent the geometry of each lever. The first lever's model space is embedded at a variable point in the transfer turret's space. This is the lever's fulcrum (labelled 'p1' in the figure). The second lever's space is embedded in the first, at the end of the lever geometry (point 'p2' in the figure). It is assumed that roller followers of a particular diameter are to be used, and their centres are at points 'f1', and 'f2' in the figure. The end-effector of the mechanism is the pocket, and its centre is located at point 'e2'.

Given a series of points on the motion path for the pockets, the following sequence of operations is performed by the macro in order to obtain a corresponding series of points on the two cam pitch curves.

1. Rotate the turret by an amount dictated by its speed, and the time increment between points on the path.
2. Assemble the mechanism. The lever model spaces are rotated until point 'e2' coincides with a point on the path. The constraint modeller's optimisation algorithm is used to do this: minimising the distance between 'e2' and the path point.
3. Plot (and store) points at the current positions of the follower centres, 'f1' and 'f2'.

An initial guess at suitable values for the lengths of the lever arms and their pivot positions, after using the above procedure, resulted in an unsatisfactory pair of cams. This was because the pitch curves clashed with the hard surfaces of the main turret (as defined by the clearance radius). Calculations of pressure angle also showed the design to be unacceptable because the peak values were well above thirty degrees.

A suitable approach taken to resolve these problems is to persevere initially with the original path, and make a more detailed investigation of the ability of this arrangement of levers to follow it. Having done this, the parametric models of the timing requirements and lever system provide a convenient means to make further tests with different profiles. These can be created using different values for the event times *t1*, *t2*, *t3*, and *t4*, or by changing the number of pockets on the transfer turret.

While making adjustments to key dimensions of the levers by hand, it became apparent that, even in this simple mechanism, there were too many variables to handle efficiently without the aid of computer-based optimisation. Because of this, the constraint modeller's optimisation algorithm has been employed to synthesise an acceptable set of dimensions for the mechanism. Eight variables, defining the position of joints, followers and the end-effector, have been declared as variables for the purposes of optimisation. In addition the constraint modeller has been given two objectives. The first is to satisfy the 'hard' constraint of clash avoidance. If this is not achieved for any particular design of levers, then the objective function accumulates a large penalty value for every point on the pitch curve that fails the test. The second part of the objective is to keep the peak pressure angle within acceptable bounds.

The pressure angle at each point on a pitch curve is estimated using the following technique. The tangent to the curve is approximated by a line drawn between the two points either side of the point in question. For accuracy, this method requires a large number of points to describe the pitch curve. In this case, 240 have been used. The pressure angle is then calculated as the angle between the tangent, and a line drawn between the follower centre and the pivot centre.

The pressure angle constraint has been encoded by accumulating an error value for every pitch curve point that has a pressure angle above a threshold value. The error is simply the difference between the pressure angle and the threshold value. The use of cumulative errors suits the constraint modeller well, because it naturally provides a 'direction' in which the direct search optimisation technique can move. Errors become progressively smaller as the system approaches an acceptable solution. A threshold value of 20 degrees has been used. Although 30 degrees is often quoted as a reasonable maximum, the more stringent requirement used here allows for errors in pressure angle due to the approximation technique used.

Other factors that determine the quality of the design relate to the size and spatial layout of the component parts. For ease of installation and maintenance it is useful if adjacent pairs of levers do not cross over each other in the plan view. The detail design of the followers is also simplified if the cam profiles do not cross.

With these design criteria in mind, the optimisation results depicted in figures 8.7-8.9 can be considered a very acceptable solution to this design problem. These are results for a transfer turret with 12 pockets, and an end-effector path where the blends from one radius to the other occur in a small space. The top blend moves slightly outside the transfer turret radius before the path follows the main turret circumference, but the bottom one just cuts inside. The pitch curves of the cams are shown, together with the path followed, in figure 8.7. The cams generated using `manually-optimised' lever geometry variables are shown in the top part of the figure. They were used as the starting conditions for the automatic optimisation performed by the constraint modeller. The result of this was to achieve the pressure angle criterion defined, and this manifests itself in the form of more circular cams (shown in the bottom part of the figure). In addition, the optimised pitch curves do not touch or cross, and the clearance with the main turret is slightly increased. A plan view of the layout of all 12 lever pairs for this design is shown in figure 8.8. There is no overlap of adjacent pairs around the greater part of the turret, with a possibility of just a small amount of overlap in the region where the transfer of cans onto the main turret takes place. One possible layout of these components in three-dimensional space is illustrated by the solid model in figure 8.9. Here the cams are at the bottom of the transfer turret, and the levers rotate about the vertical shafts.

In order to investigate more fully the capabilities of the cam/lever concept, the optimal lever geometry described above was used with a motion profile which has blended regions passing well outside the radius of the transfer turret. Initial attempts to optimise this system resulted in the rocker arms of the cam followers shrinking to zero length in order to satisfy the pressure angle constraint. An additional constraint was therefore required to prevent this happening. It was encoded in such a way that a penalty function was applied if the length fell below a threshold value of 30mm. This is a reasonable figure given the need, in the embodiment phases of the design process, to house bearings at the pivot points, and to use a roller follower of about 25mm diameter. It also ensures reasonable leverage characteristics in operation. With this additional constraint the optimisation procedure yielded the geometry in figure 8.10. It is evident from the figure that, in order to satisfy the pressure angle condition, the lever length and clearance conditions are only just met. In other words, the outer cam has grown, the inner cam has shrunk, and the rocker arms are set to the minimum length allowed by the constraint.

Similar characteristics were found when synthesising geometry for designs incorporating just nine pockets on the transfer turret, regardless of whether the end-effector motion was specified to pass outside or inside the turret radius within the blends. Optimisation results for these designs are shown in figures 8.11 and 8.12. Those in figure 8.12 are unacceptable from either clearance conditions (top half of figure) or pressure angle (bottom half of figure). However, the optimised cams in figure 8.11 (lower part) are satisfactory. They have greater clearances with the main turret than the twelve pocket design in figure 8.7, but the rockers (not illustrated) are at the minimum length allowed.

These results are interesting because it seems at first there might be scope for a more simple layout of levers when there are fewer pockets on the transfer turret. However, the potential advantage of this is not possible in practice, because the radius of the turret has to be reduced in order to match the circumferential speed of the main turret.

Summary of constraints

- follow the main turret radius, matching the main turret's speed, during the loading phase (from t1 to t2).
- follow the transfer turret radius at the required speed when returning to pick up a new can (from t3 to t4).
- curve fitting to generate the blends from one circle to the other constrained by setting velocity, acceleration and jerk values at the segment boundaries to match those of each sine wave.

Optimisation

- obtain new path shape to test by altering t3 and t4, and re-fitting the blending curves.
- use optimisation to investigate whether acceptable lever geometry can be obtained to follow the new path.
    - constrained by pressure angle, clash avoidance, and minimum lever length requirements.
    - design variables: lever lengths, pivot positions relative to transfer turret centre, and number of pockets.

8.4 Effect of path shape on actuator kinematics

In this section, attention is turned briefly to an alternative design concept, that of using actuator pairs to guide the transfer pockets, rather than the cam-driven levers (cf. figure 8.1c). Again the motion profiles designed with the timing diagrams are used to drive a kinematic model of the system, thereby determining the input motions needed for the actuators.

The machine model in this case is considerably simpler than that used for the cam and lever design concept. It consists only of two model spaces: one for the transfer turret, and one for the linear actuator. The linear actuator is represented by a line of varying length, drawn from the turret centre to a point on the designed pocket locus. Hence the centre of rotation of the actuator relative to the turret is at the turret centre. To model the operation of this machine it is only necessary to increment the angle of the transfer turret, and redraw the line. The actuator length and angle relative to the turret are then stored, so that angular and linear velocities and accelerations can be calculated. Figure 8.13 shows a stick diagram of the design for twelve actuator pairs.

This concept also allows more scope to vary the radius of the pocket locus. This is because larger differences in tangential speed between the two turrets can be accounted for by the motion of the actuators. The disadvantage of this however is that the more the actuator is required to move, the greater is the power consumption of the machine. This is compounded by the fact that the linear actuator must cater for acceleration effects caused by the rotation of the turret and the rotary actuator. A further disadvantage is the complication of fitting drives for large numbers of actuators on the rotating turret.

Despite having these drawbacks, some kinematic properties for the design have been calculated. There is relatively little difference between the levels of velocity and acceleration when the two different path shapes in figure 8.13 are used. The path that runs inside the transfer turret locus during the transition causes slightly higher actuator accelerations, but the peak velocities are smaller.

8.5 Concluding remarks

The process of optimising motions together with mechanisms has been rather more involved in this case study, with a requirement for 'nested' optimisation which may be partly manual, and partly automated. There is a significant amount of work required to investigate whether a given motion profile can yield an acceptable mechanism, so it is not appropriate to produce large numbers of instances of the motion.

It is interesting to note that it was not necessary to encode constraints specifically to stop the pitch curves touching or crossing. The fact that the optimisation moved the design away from this tendency is, in part, a consequence of applying the criterion to improve pressure angles. This shows the value of using the constraint modeller in a progressive way: applying a small selection of critical constraints at the outset, then examining the graphical results to see how the characteristics of the design have changed. It is not necessary for the designer to use thought experiments to try to construct an exhaustive list of constraints before presenting the problem to the constraint modelling system.

The work carried out on the cam and lever design suggests that the concept is basically feasible, although it would require careful manufacture and maintenance to enable it to operate accurately. This is because of the tolerances in manufacture, and the prospect of wear on the cam surfaces and in all the pivot bearings. Also the detail design of the levers must ensure a sufficiently stiff structure to minimise displacement errors due to deflection. This is of concern because the 'packaging' requirements of the machine make it necessary to mount the cams well below the plane of the main turret pockets. The most convenient arrangement may be to design the vertical offset of the levers along the axis of rotation. This would require the use of two bearings for each lever (one close to the cam followers, and one close to the pockets), but would increase stiffness.

The design concept using actuator pairs to guide the pockets has been investigated to illustrate the use of an alternative mechanism type with the same motion data. Its application to this problem is rather academic however, because the energy required by the actuators would be significant, even though they are only required to make small adjustments to the pocket displacement.

A further aspect of the design approach taken in this case study is its relationship with the ideas of recording 'design history' and 'design intent'. The initial work on the design centred around an early decision on the angle of overlap between the transfer turret and the main turret. However the provision of the overlap is only made as a means of satisfying what might be regarded the 'real' constraint in the problem: namely, providing enough time for the transfer operation to take place. Obviously the two are directly related, but the point here is that the design process and documentation ought to record the constraint as one of time rather than angle of overlap, because this is a better expression of the 'design intent'. This reduces the risk of misinterpretation of the design decisions if any further work is carried out on the machine by other designers. Motion profiles might, for example, be regenerated that keep the same angle of overlap, but which cut inside the transfer turret radius to such an extent that the time constraint is (unwittingly) violated. The constraint modelling design approach is useful here, because it encourages the designer to consider the motion requirements in detail.

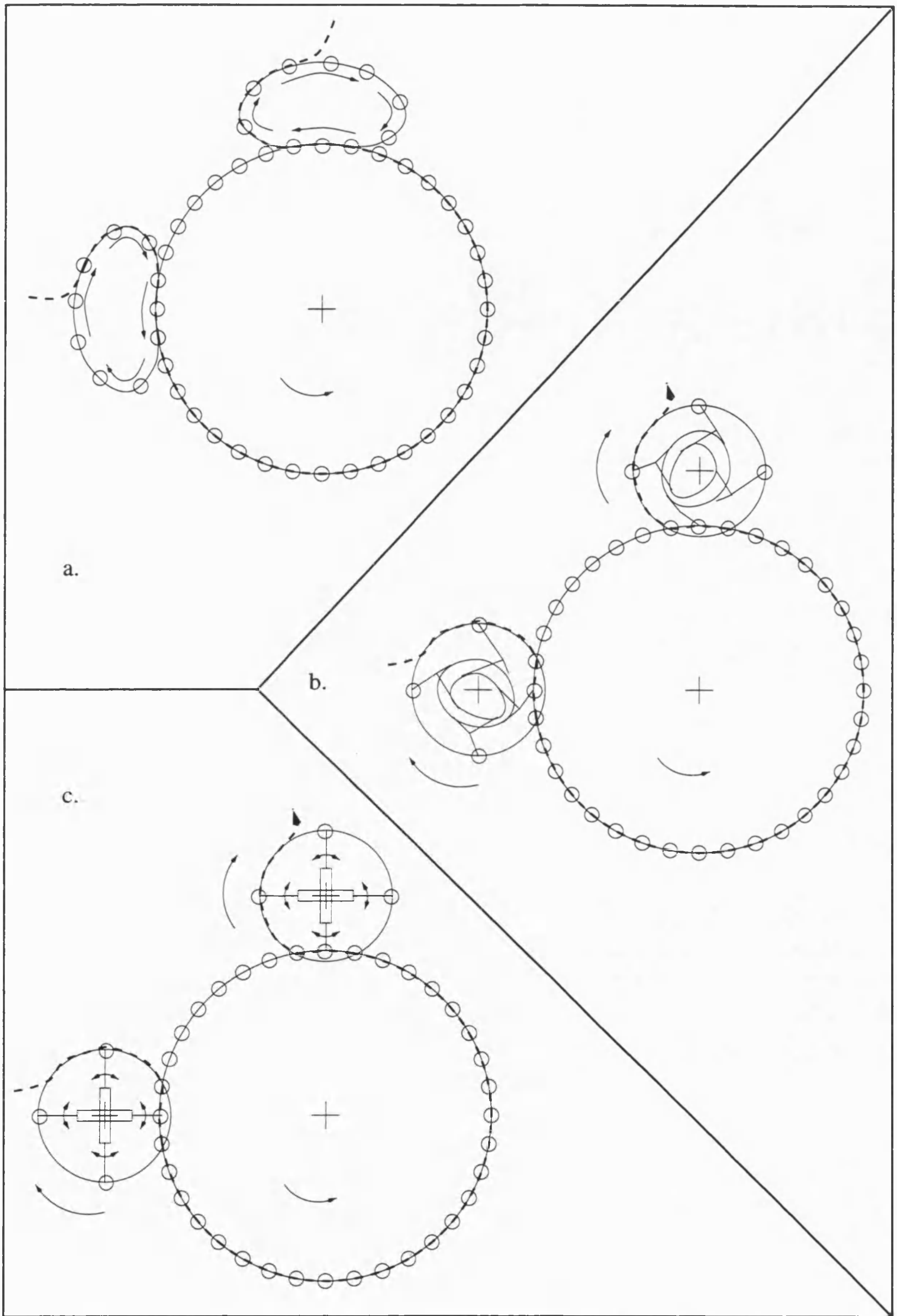


Figure 8.1 Design concepts

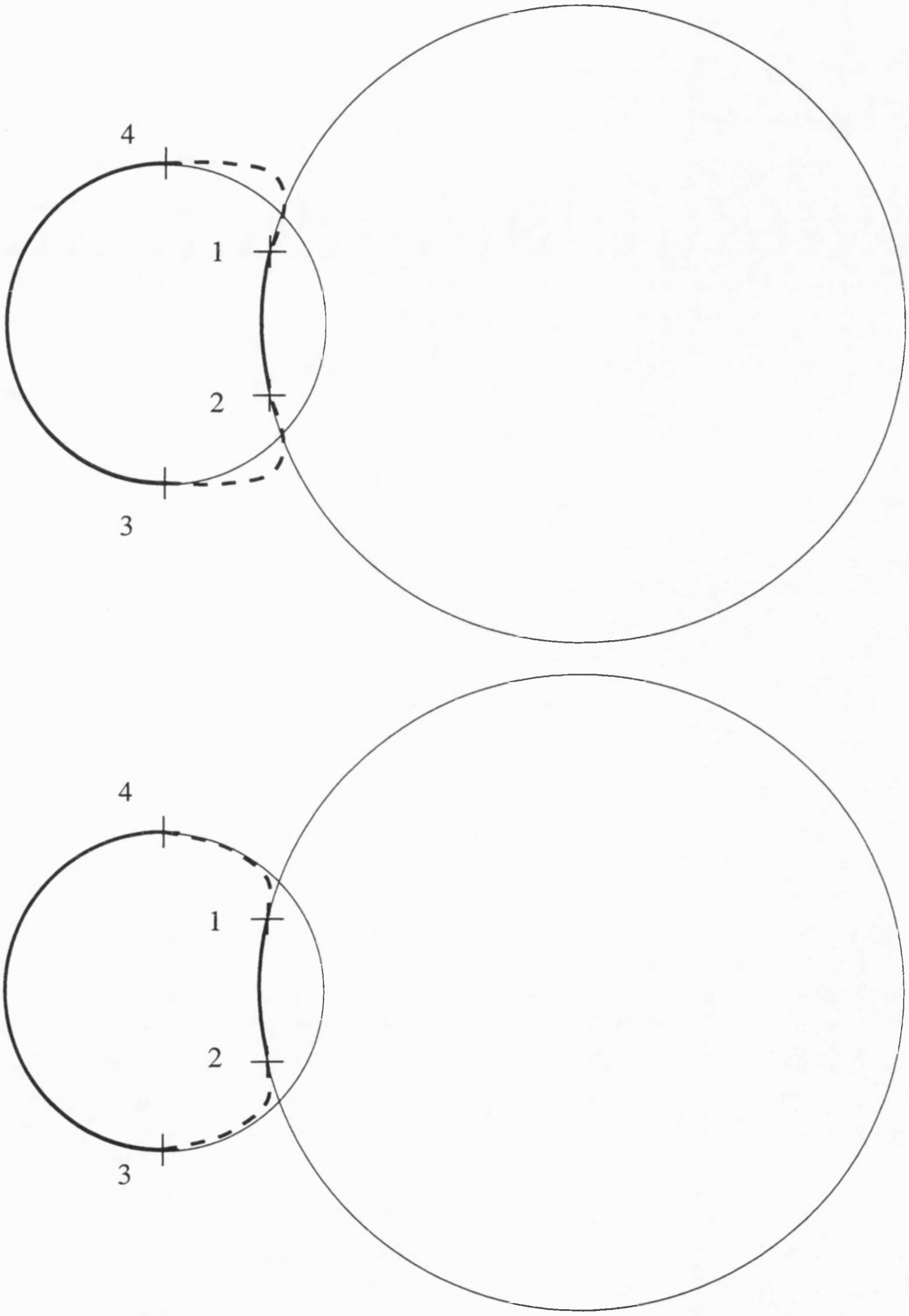


Figure 8.2 Transition segments passing outside or inside the transfer turret radius

Summary of displacement constraints

A. follow the main turret radius, matching the main turret's speed, during the loading phase (from t1 to t2).

B. follow the transfer turret radius at the required speed when returning to pick up a new can (from t3 to t4).

Other motion constraints

- duration of loading phase (t1 to t2) determined by time required by main turret to grab a can.
- curve fitting to generate smooth blends from one circle to the other constrained by matching velocity, acceleration and jerk values at the segment boundaries.

Optimisation

- obtain new path shape to test by altering t3 and t4, and re-fitting the blending curves.
- use optimisation to investigate whether acceptable lever geometry can be obtained to follow the new path.
    - constrained by pressure angle, clash avoidance, and minimum lever length requirements.
    - design variables: lever lengths, pivot positions relative to transfer turret centre, and number of pockets.

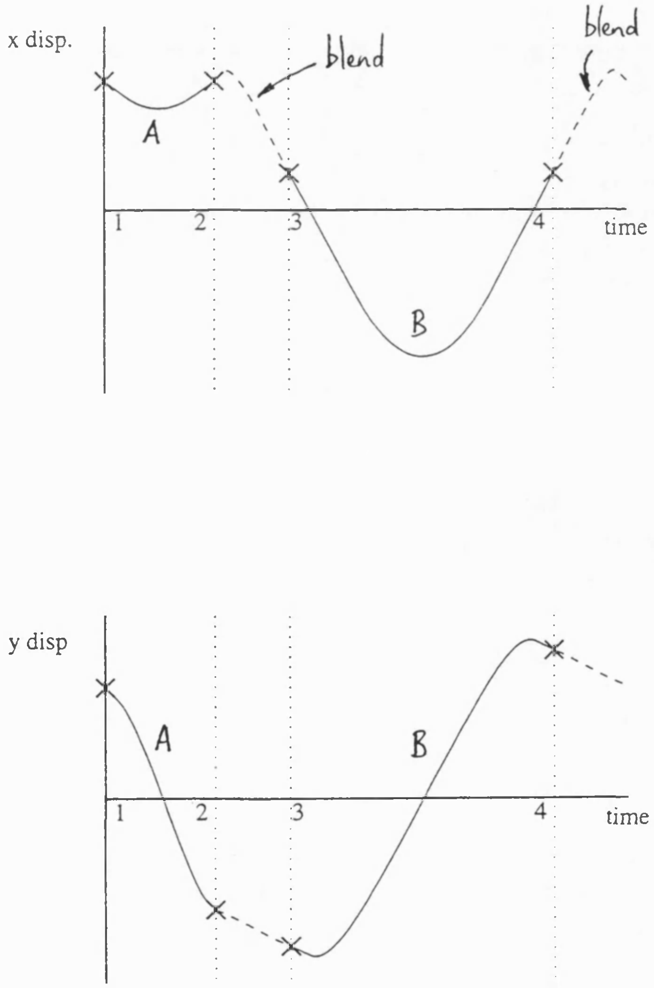
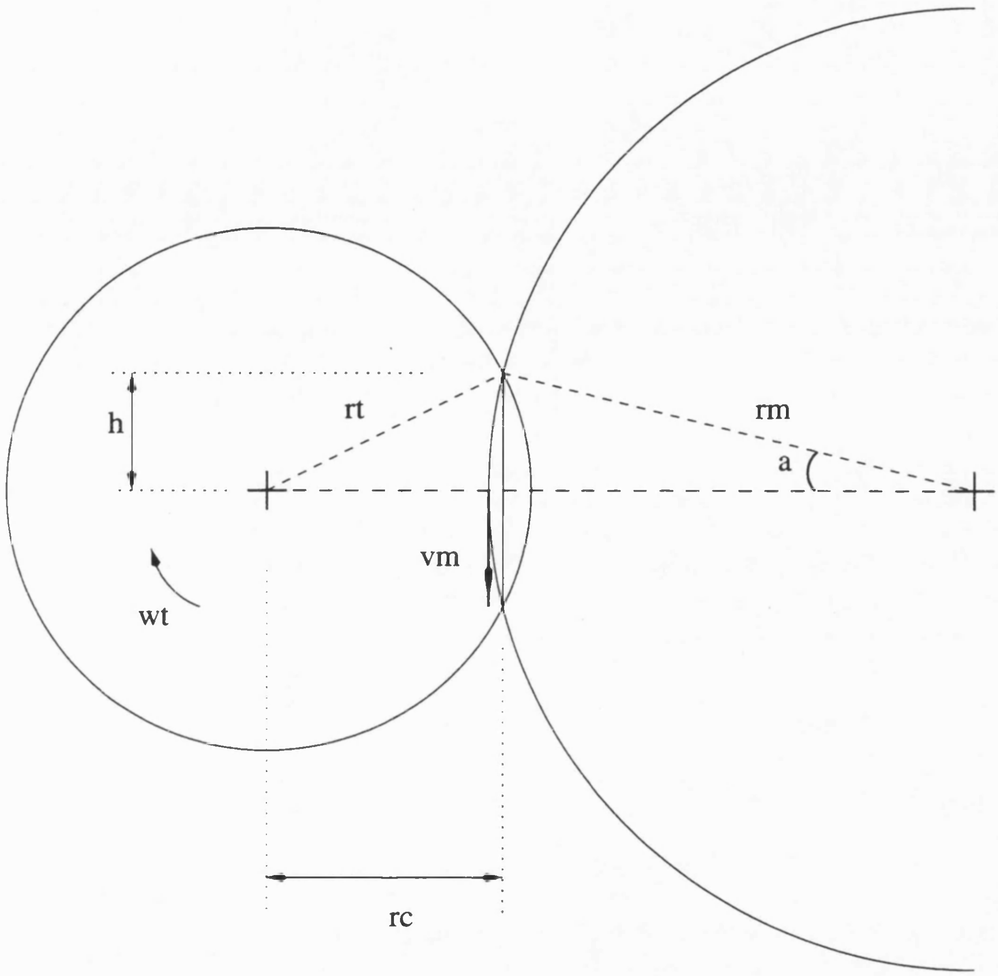


Figure 8.3 Parametric timing diagrams



to set rt so that speed at rc equals that at main turret circumference:

rc = vm/wt

h = rm*sin(a)

rt = sqrt(rc*rc + h*h)

Figure 8.4 Selection of transfer turret radius

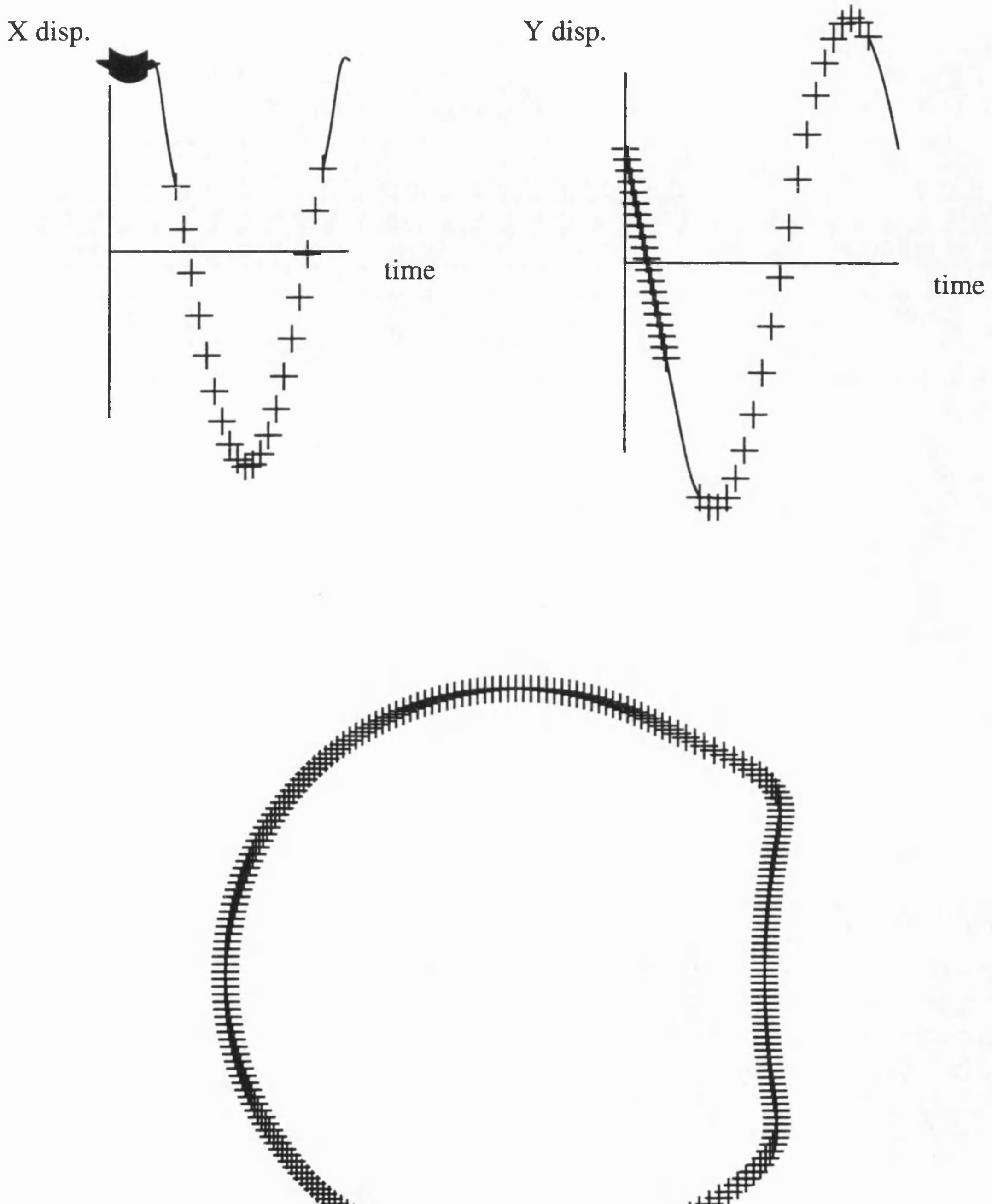


Figure 8.5 Example path

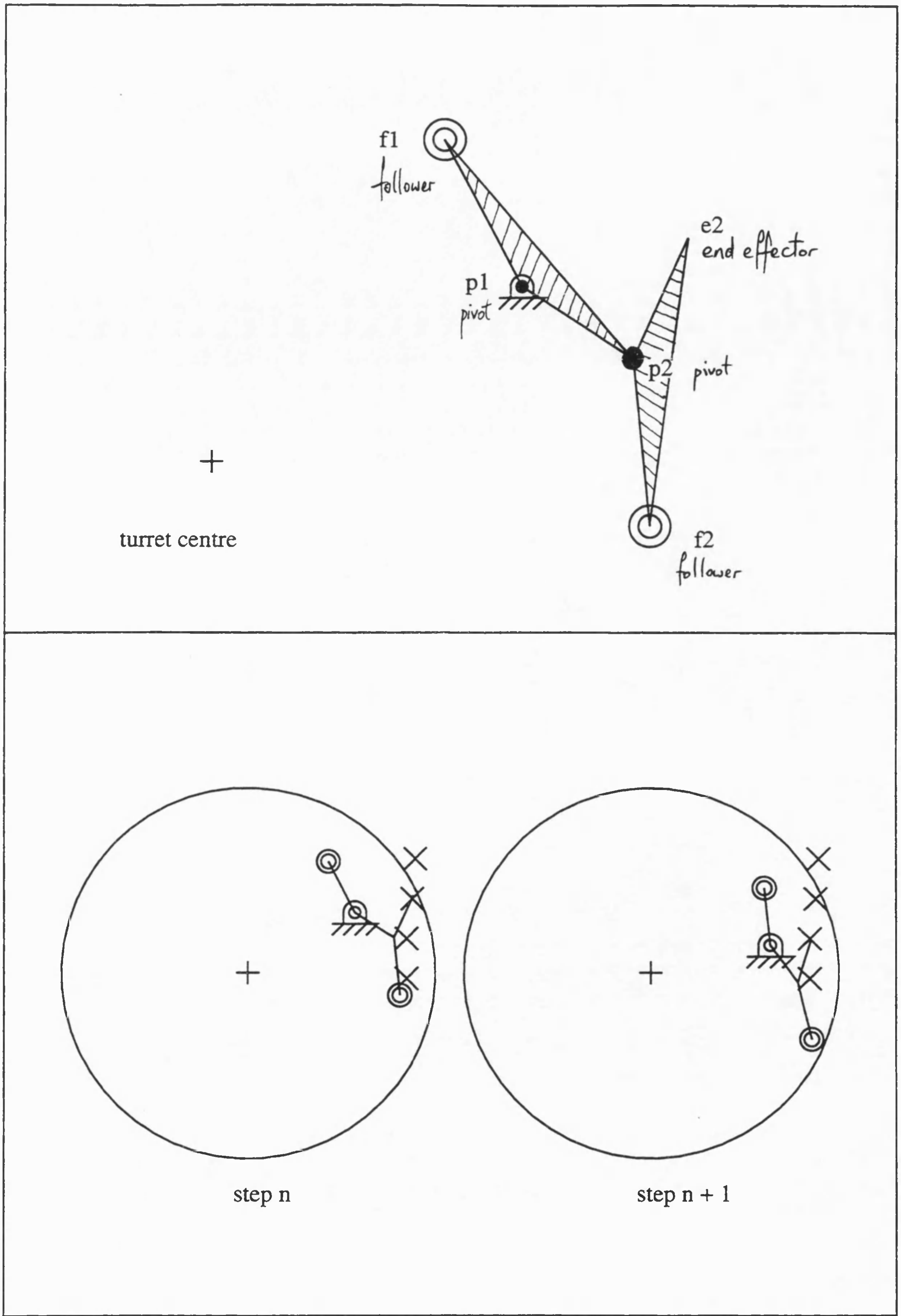
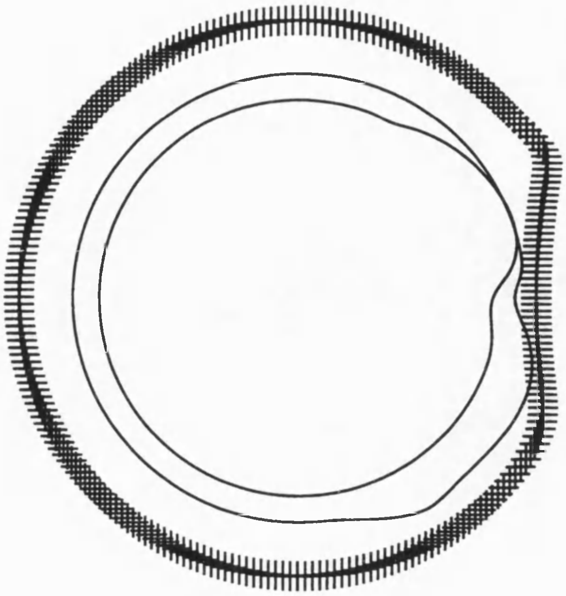
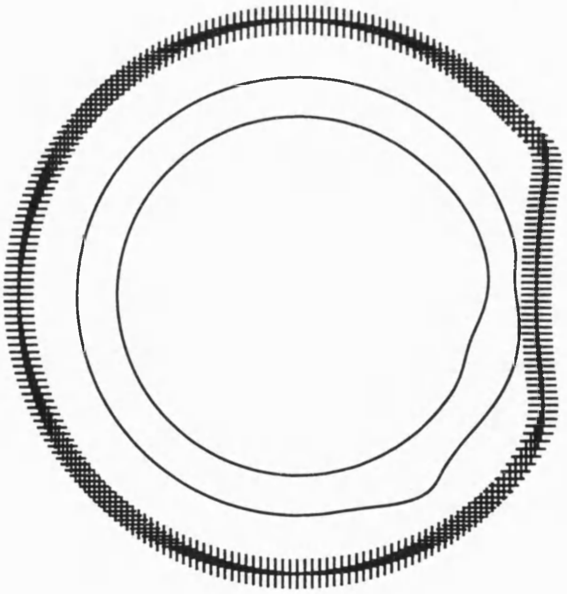


Figure 8.6 Cam-following levers and pitch curve generation



Before Peak pressure angle: 45 degrees (inner cam)
39 degrees (outer cam)



After Peak pressure angle: 18.5 degrees (inner cam)
20 degrees (outer cam)

Figure 8.7 Cams for the 12 pocket design, before and after optimisation

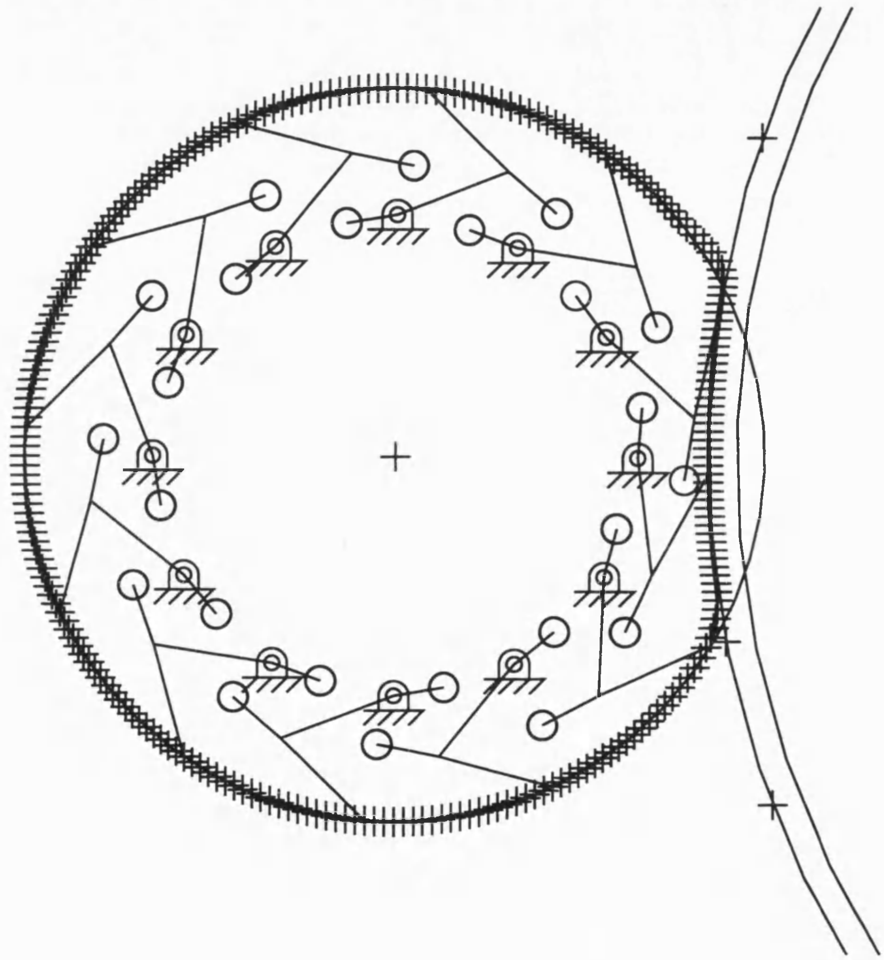
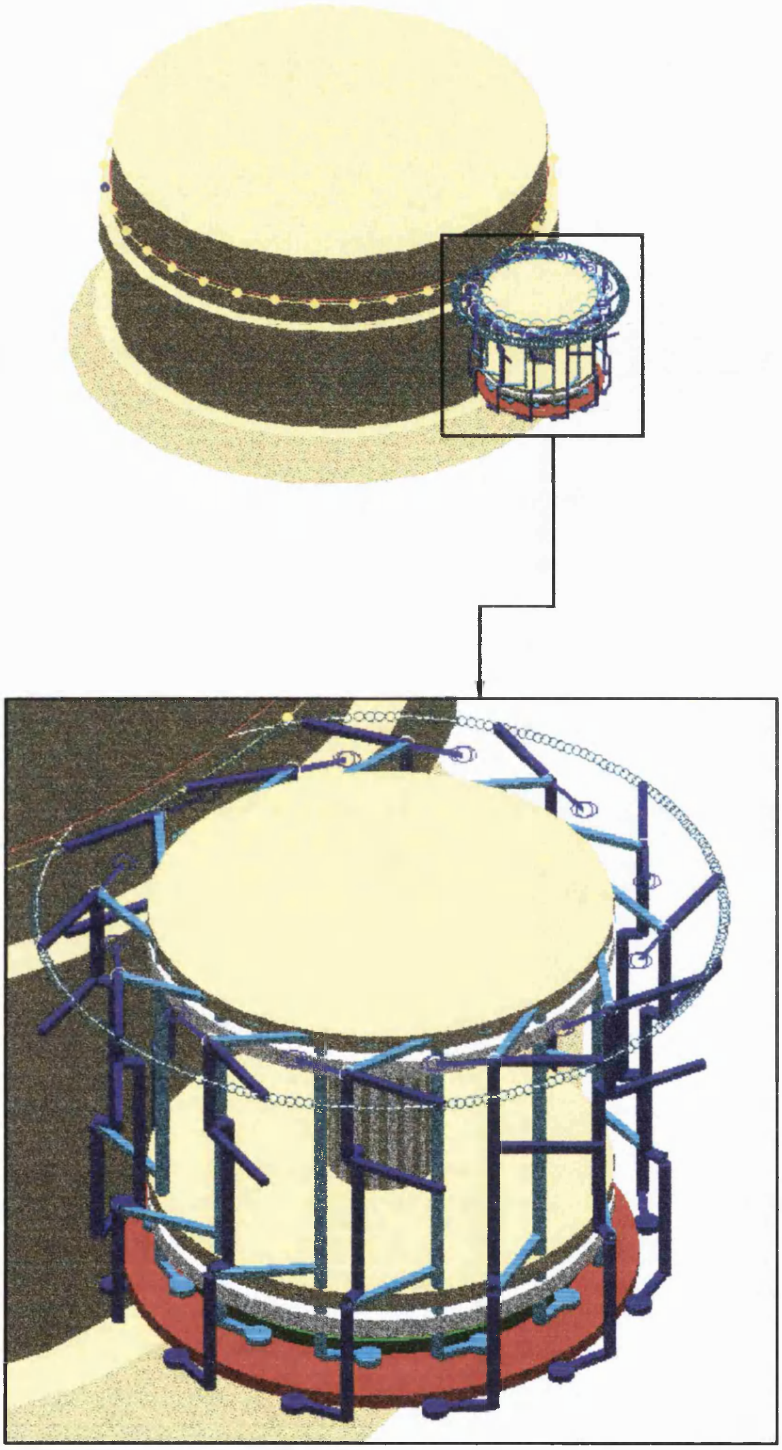
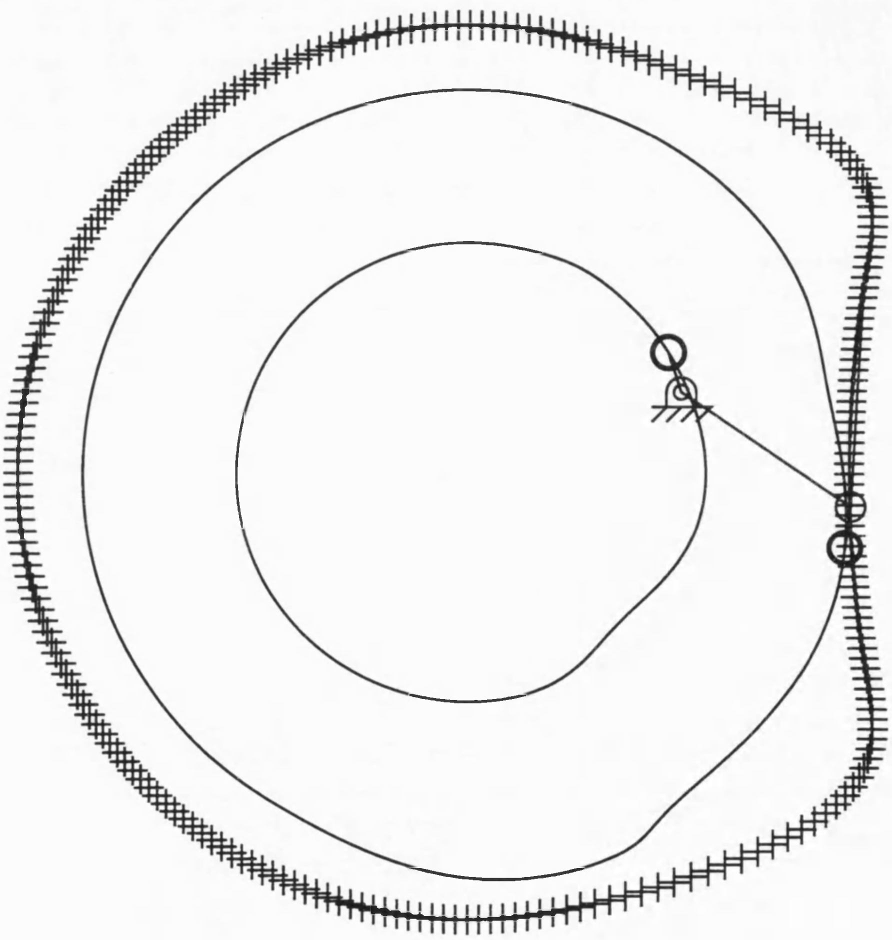


Figure 8.8 Optimised lever geometry for the 12 pocket design



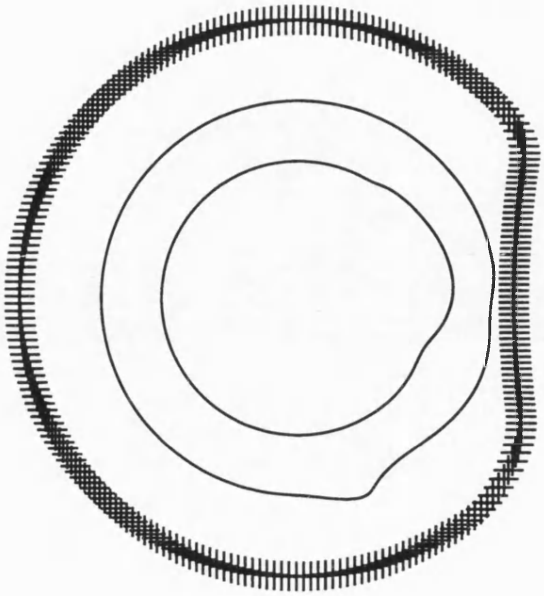
Cams shown in red and green at the base of the transfer turret

Figure 8.9 Solid model of the 12 pocket design

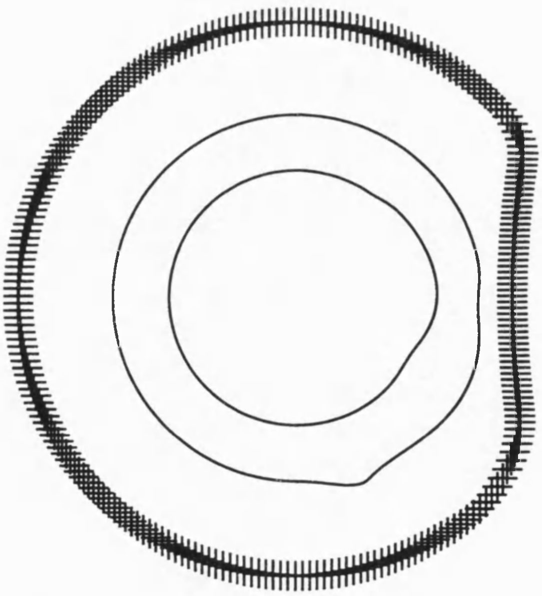


Peak pressure angle: 18.6 degrees (inner cam)
20 degrees (outer cam)

Figure 8.10 Synthesised cam geometry for path blend outside turret radius
(12 pocket transfer turret)



Before Peak pressure angle: 29 degrees (inner cam)
30 degrees (outer cam)



After Peak pressure angle: 19.5 degrees (inner cam)
20 degrees (outer cam)

Figure 8.11 Cams for the 9 pocket design, before and after optimisation

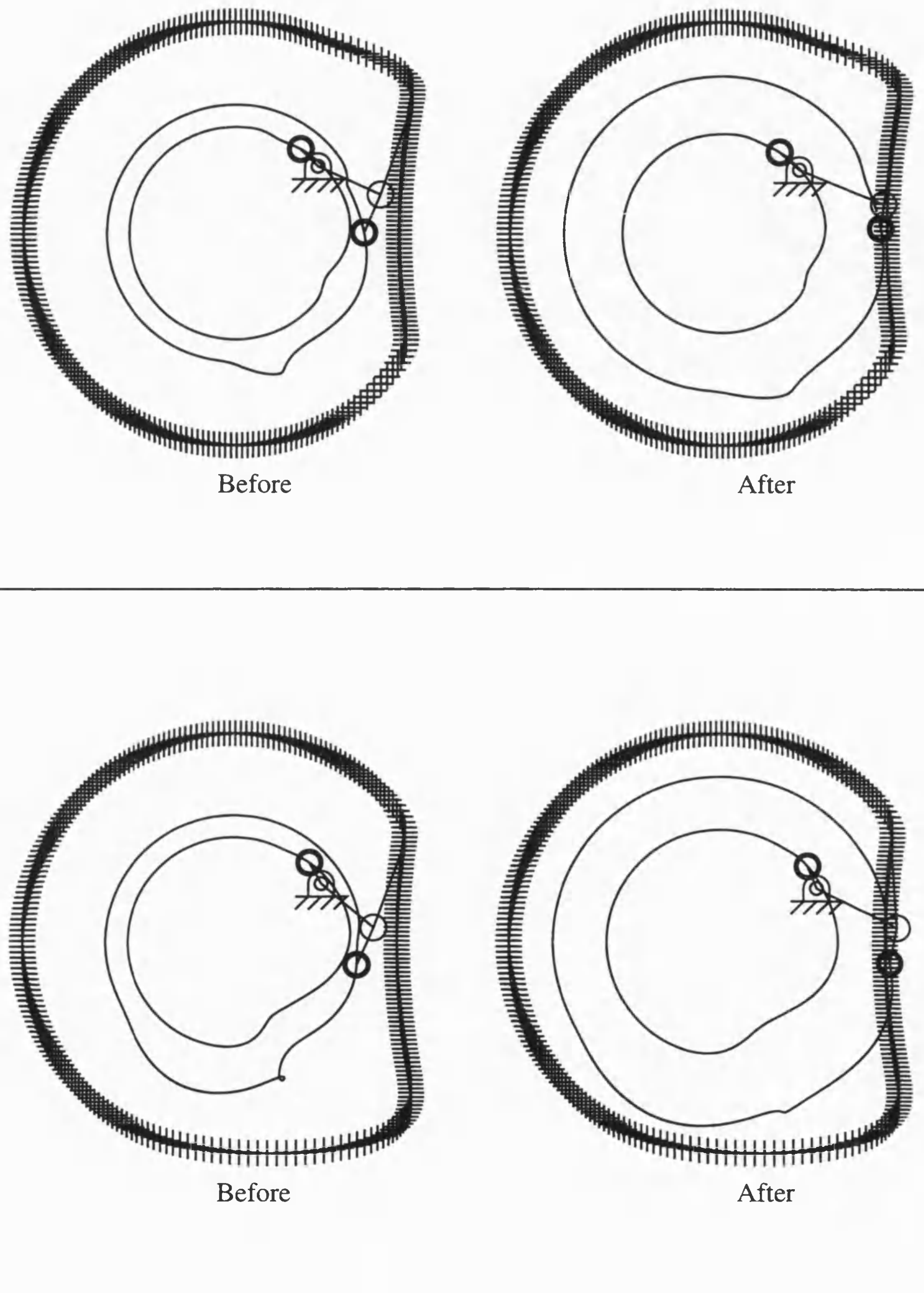


Figure 8.12 Optimisation of cam and lever geometry for various paths
(9 pocket transfer turret)

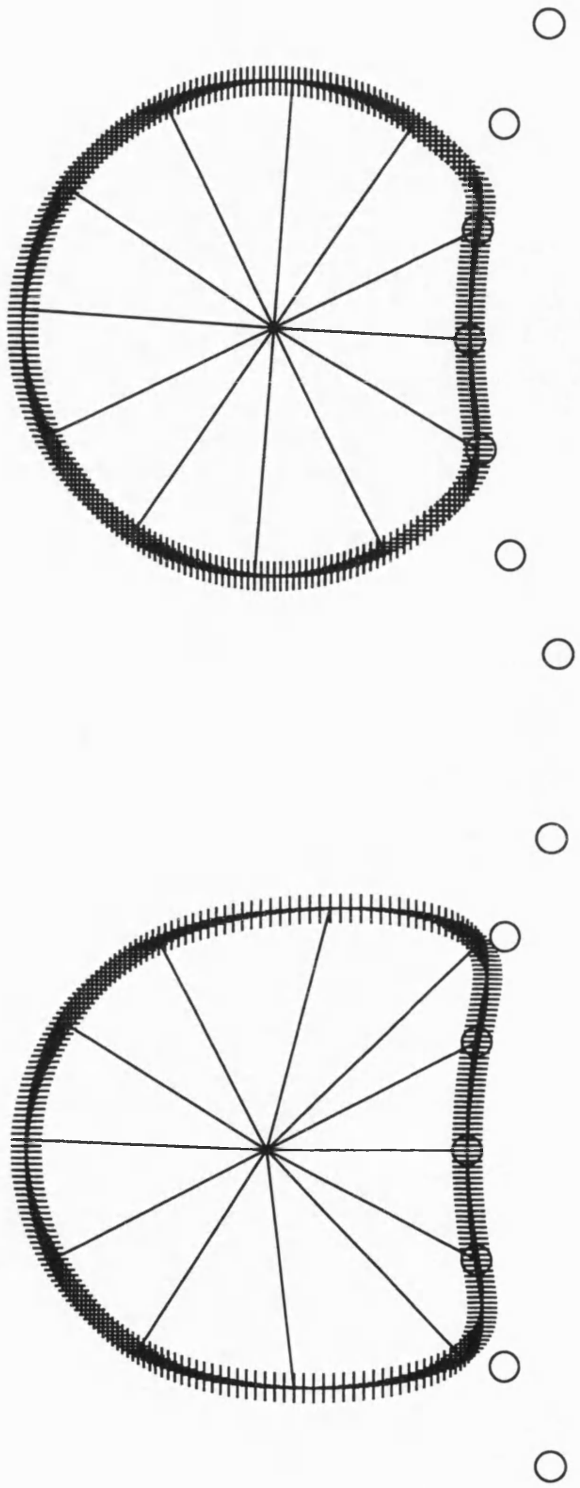


Figure 8.13 Actuator-driven transfer pockets

Chapter 9 Case study 6 - pushing boxes into a labelling machine

This case study concerns the design of a device to load boxes into a labelling machine, as described in chapter 3. The method of approaching the problem of designing a suitable motion for this device is described here.

9.1 Design concepts

A number of potential concept designs exist to achieve the task of loading the labelling machine with boxes. The schematics in figure 9.1 illustrate three of the possibilities, the last of which is treated in more detail for the purposes of this study.

The first, shown in part a) of the figure, is a conveyor belt that has an intermittent motion. It is initially at rest when a box is loaded onto it from the perpendicular 'supply' conveyor. It then experiences a dwell-rise-dwell motion so that it accelerates the box towards the labelling machine, decelerates and eventually comes to rest again when the box is inside the machine. This system could be driven by a stepper or servo motor, and would be a compact, space-efficient design. It would also be relatively simple to implement, and to reprogram for changes in production rate. The constraints that would need to be taken into account are as follows.

- the maximum acceleration levels the product inside the box can tolerate.
- the limit on deceleration imposed by friction between the box and the conveyor.
- the maximum throughput rate of the labelling machine.
- the positional accuracy required of the 'final' position under the labelling head.

A disadvantage of this solution however is relative expensive, both in terms of the cost of the hardware, and the energy consumption associated with continual periods of acceleration and deceleration during the machine cycle.

The second concept (figure 9.1b) is a simple linear actuator that pushes the box along a stationary, horizontal chute into the labelling machine. A drawback of this design however is the need for the actuator to return to its initial position before the next box can be moved onto the chute. This limits the maximum throughput rate of the whole loading system.

The third concept, shown in part c) of figure 9.1, is a planar mechanism that traces out a looped path. During the first part of the motion it pushes the box along the chute, and on the second part it returns. During the return it loops over the next box, which has already been placed at the start of the chute ready for the next cycle. Obviously there is a need in such a layout to position the mechanism in a vertical plane alongside the conveyor in order to avoid clash. The end-effector of the mechanism then reaches across the conveyor belt to contact the box.

Assuming the time needed to position a box at the start point is very small, the maximum throughput rate for this system depends on the cycle time of the mechanism alone. However, the motion during the cycle is obviously subject to similar constraints regarding acceleration and deceleration of the box as in the first concept. Another motion constraint may arise if the impact speed of the mechanism end-effector as it strikes the box has to be limited to a certain value. This may be to avoid damage or to ensure the box does not accelerate away from the end-effector due to the elasticity of the impact. The motion constraints for this system are discussed in greater detail in the next section, where their effect on suitable motion profiles is examined. Clearly this design concept is subject to more motion constraints than the first two, as the resulting motion is two dimensional, rather than one dimensional.

At first sight it appears that the existing path-matching synthesis techniques described earlier might be sufficient to tackle this problem. However, after sketching a number of suitable paths and using the mechanism selection program, it becomes apparent that more control has to be placed on timing around the path than is possible with a uniformly driven linkage. The constraint-based timing diagrams are therefore a convenient tool for the generation of both path and timing for the loading device.

9.2 Construction of the parametric timing diagrams

As with the other case studies, the process of constructing the parametric timing diagrams begins by considering, with the aid of the sketch, the constraints on the horizontal and vertical components of the motion. In this case however, a larger number of arbitrary constraints are required, and a closed curve is used to interpolate the motion in each direction over the whole cycle. Events of interest during the cycle of the mechanism are labelled in figure 9.2. They are described below.

- Event 1: the end-effector hits the stationary box.
- Event 2: the box is deposited inside the labelling machine.
- Event 3: the end-effector clears the overhanging geometry of the machine.
- Event 4: the end-effector begins to pass over the next box.
- Event 5: the end-effector clears the box.

The horizontal displacement at each of these events is specified by the basic geometry of the system. In this instance the box is 100mm long, 60 mm high, and has to be pushed 500mm along the chute. The front face of the box has to travel 200mm into the labelling machine, so, given the length of the box, the end-effector has to return under 100mm of overhang before it is clear of the machine. The precision points for the horizontal displacement are therefore as follows, given that the origin of the *xy* space is taken to be on the surface of the chute, at the start of the cycle.

$x1 = 0$

$x2 = -500$

$x3 = -400$

$x4 = -100$

$x5 = 0$

The technique used to insert these constraints on the diagrams is to use arbitrary time variables for each of the events (except event 1, which by definition occurs at $t1=0$). Along with these values, corresponding initial values for $y1, y2, \ldots, y5$ are chosen. Each of these selections is bounded by the constraints imposed by clash avoidance.

Two precision points also have to be inserted on the horizontal velocity diagram. At the start of the cycle the horizontal speed of the mechanism end-effector is constrained to lie within a range of permissible speeds. Experimentation with the boxes would be used to determine the maximum speed of impact allowed. The second precision point required is at time $t2$, where the horizontal speed must be zero. This constraint is used to ensure the mechanism does not try to push the box too far into the machine.

It is possible to remove some of these arbitrary variables by processing the x and y diagrams sequentially. In order to do this a curve is fitted on the x-t plane that passes through points $x1$ and $x2$ at arbitrary times $t1$ and $t2$. The times $t3$, $t4$, and $t5$ can then be obtained directly, and used to insert corresponding precision point constraints on the y-t plane. However, doing this removes a convenient means of manipulating the shape of the interpolated curve, so the use of the extra points is justified.

9.3 Refinement of the diagrams

Closed curves are fitted through the timing diagrams described above, to interpolate the motion over the whole cycle. However, the parameterisation of these curves has to be considered carefully. An initial trial fitting, using the minimum number of curve segments over the cycle, results in unsuitable motion profiles. The three problems encountered, illustrated in figure 9.3, are listed below.

- Oscillation of the x-t curve just after the start of the cycle causes the path to wander above the x-axis and then cause a second 'impact' with the box when it returns.
- Another oscillation occurs on the y-t curve and causes the path to dip below the height of the conveyor (as in the figure), or to rise above the height of the box. These conditions either cause clash with the conveyor, or result in the end-effector losing contact with the box.

- At the end of the cycle the path does not move far enough to the right of the next box. The physical dimensions of the mechanism end-effector would make interference with the box inevitable.

A remedy to these problems is to increase the number of curve subsegments used, and to include an extra precision point constraint between event 5 and the end of the cycle. The number of subsegments is increased so there are 3-4 knots between each precision point. Clearly the exact number varies when changes are made, either manually or automatically, to the time values of each of the precision points. The additional precision point, $(t6,x6,y6)$, is shown on the x and y displacement diagrams in figure 9.4. By fixing $x6$ at an appropriate value, the path is pulled further to the right, and this prevents the end-effector from sliding down the rear face of the box.

9.4 Optimisation of the motion profiles

The use of so many arbitrary precision points to describe the path makes it essential to use the constraint modeller's optimisation algorithm. This allows the system to test large numbers of positions for each point in order to smooth out the path. In this example there are ten degrees of freedom manipulated by the system. These are the time and vertical displacements at events 2,3,4,5, and 6. A suitable objective is to minimise the peak resultant acceleration. However, it is also necessary to include a 'penalty' function that discourages the algorithm from choosing unfeasible values for the time variables. Thus, at every trial in the optimisation, the peak acceleration is only used in the objective function if the following condition is satisfied, otherwise the penalty value is used.

$$t2 < t3 < t4 < t5 < t6$$

The path resulting from the initial 'guessed' values of the arbitrary variables is shown in figure 9.4, and this is followed by an optimised motion in figure 9.5. It is notable that the form of the motion in each direction is basically a set of intervals of approximately constant velocity joined by smooth transitions of short duration. This effect results from the large number of subsegments used in the curve fitting.

Summary of the constraints

X direction

- locate initial position of box
- locate final position of box
- location of the overhang
- impact speed
- end-effector to be at rest at final position of box

Y direction

- locate end-effector on the box (at times t1,t2, and t3)
- rise above the next box on the return

Optimisation

- obtain a smoother motion than that produced with guessed initial values for the timing and y-displacements.
- achieved by minimising peak acceleration by changing time and y-displacement values.

9.5 Implementation of the designed motion

Two systems could be devised to implement the motions created with the timing diagrams. These are a cam-driven system, and a linkage mechanism similar to that used in the ice-cream cutter case study.

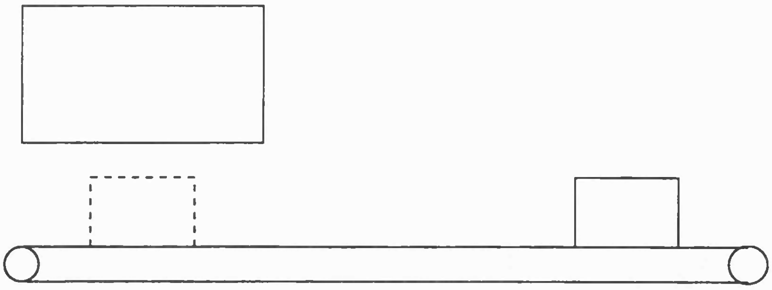
The cam-driven system requires two cams: one to provide the horizontal motion, the other to provide the vertical motion. Examples of this type of design already exist in the food packaging industry.

Figure 9.6 shows four-bar and five-bar linkages that approximate the shape of the path designed by the timing diagrams. These have been synthesised with the CAMFORD mechanism selection application. It is notable that many of the mechanisms proposed for this task by the program trace a figure-of-eight path, rather than one with a cusp. For this reason it might be considered that the timing diagrams do not provide such a useful synthesis technique. However, their timing at the start of the cycle cannot be greatly influenced, even by the use of timing dependent search techniques in the CAMFORD program. It is therefore reasonable to search the linkage catalogues for a mechanism that matches the shape of a path created with the timing diagrams, and then use the timing adjustment technique described in chapter 7 to satisfy the velocity constraints imposed in this case study.

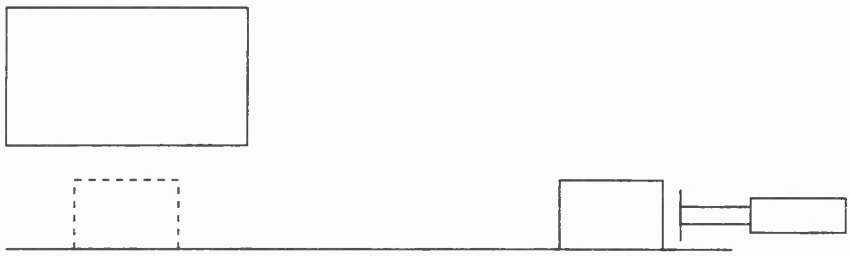
9.6 Concluding remarks

As with the other examples, the parametric form of motion description is useful because the path and timing requirements are generic in nature, and it allows a variety of instances of the motion to be produced with ease. In this case different motions could be generated to account for variation in box sizes, impact velocities, and friction properties between box and chute. The construction of the diagrams also gives the opportunity to investigate the effect of reducing cycle times, as in the case of the ice-cream cutter.

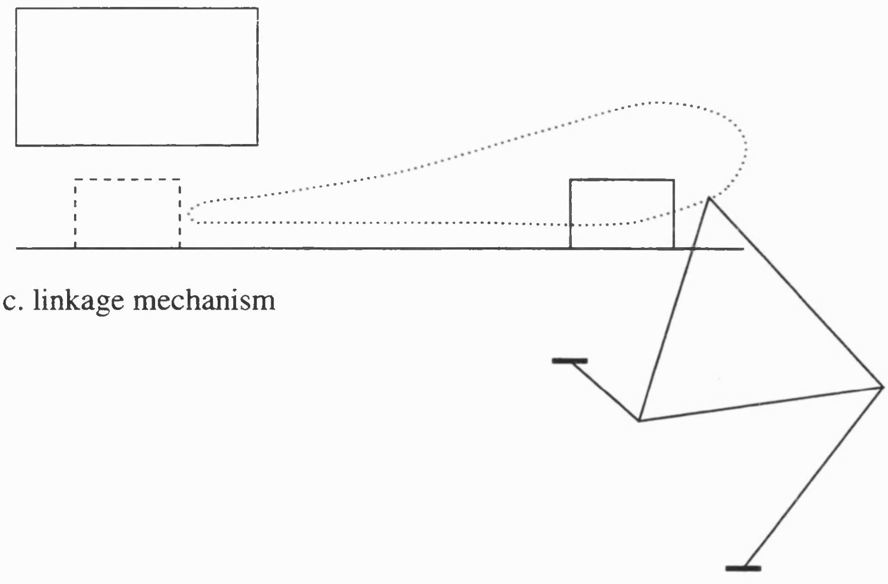
Unlike the other case studies however, this one has involved the use of a large number of arbitrary precision points. It is difficult to choose values manually for these that result in a smooth profile, and the use of optimisation has been essential to perform the smoothing.



a. conveyor belt with programmed motion

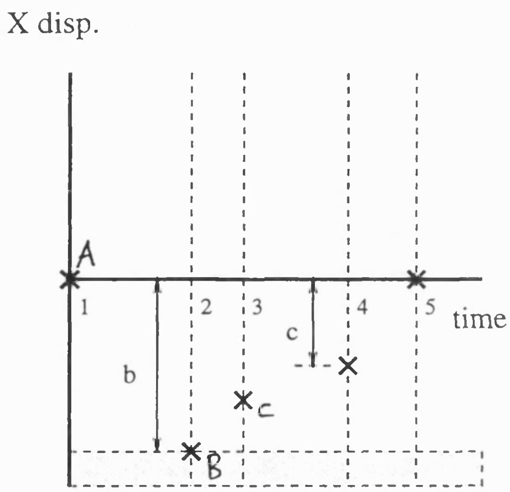


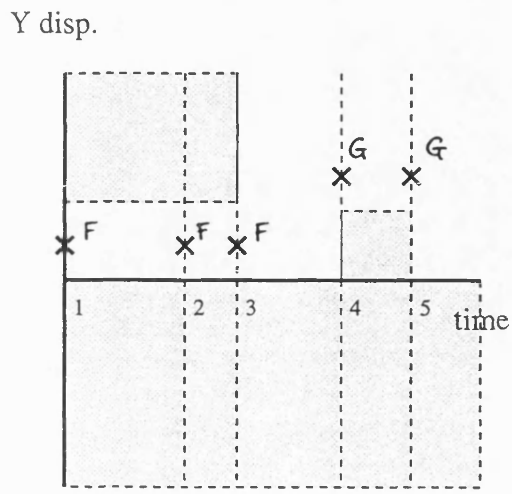
b. linear actuator

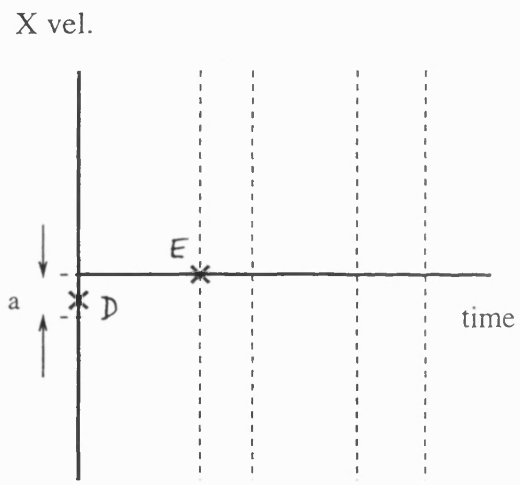


c. linkage mechanism

Figure 9.1 Design concepts







Notes

a - range of allowable impact speeds

b - stroke length

c - box length

shaded areas must be avoided

<u>Summary of the constraints</u>

<u>X direction</u>

A. locate initial position of box
B. locate final position of box
C. location of the overhang
D. impact speed
E. end-effector to be at rest at final position of box

<u>Y direction</u>

F. locate end-effector on the box (at times t1,t2, and t3)
G. rise above the next box on the return

<u>Optimisation</u>

- obtain a smoother motion than that produced with guessed initial values for the timing and y-displacements.
- achieved by varying time and y-displacement values to minimise peak acceleration.

Figure 9.2 Parametric timing diagrams

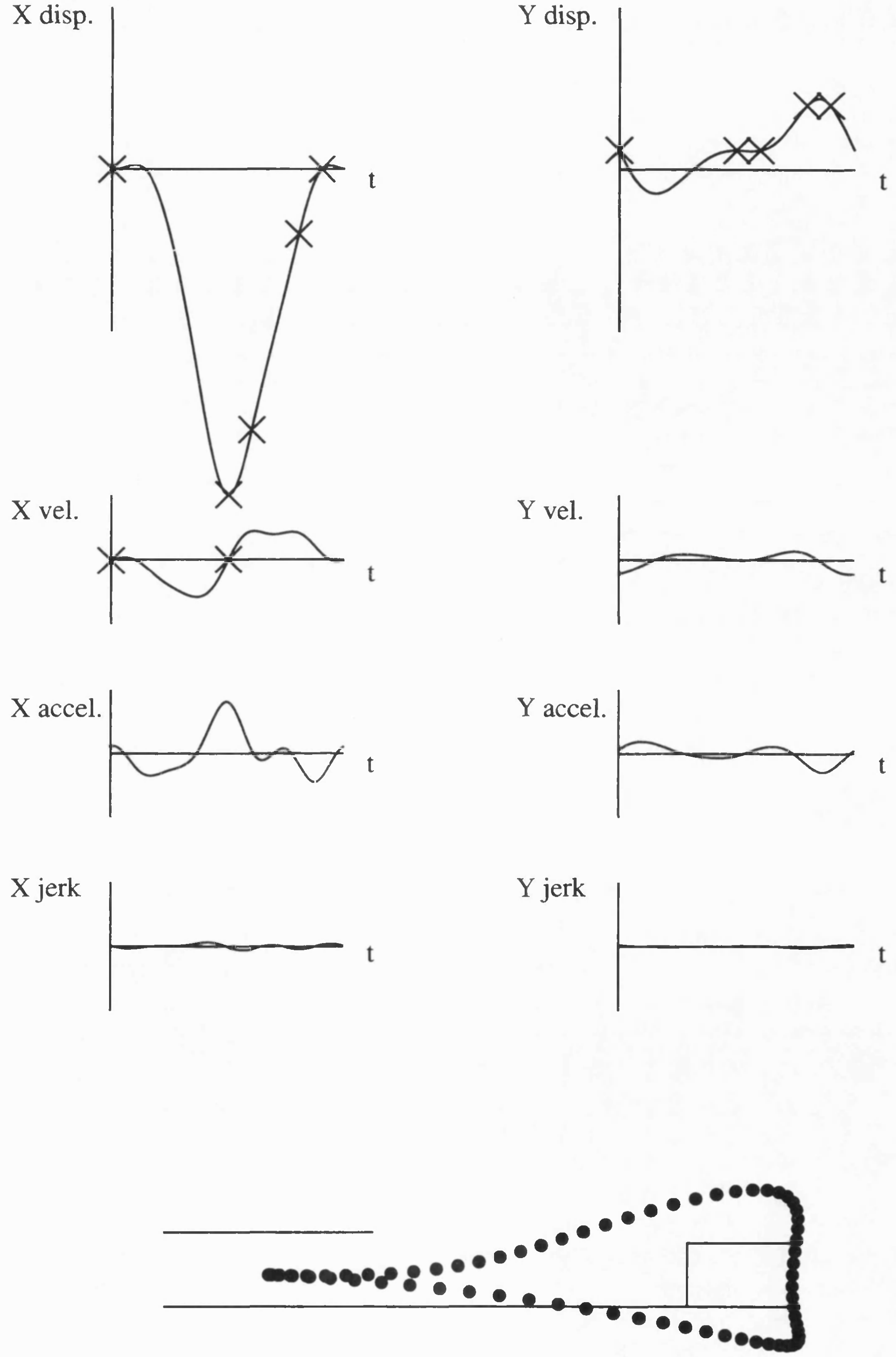


Figure 9.3 Problems with curve fitting

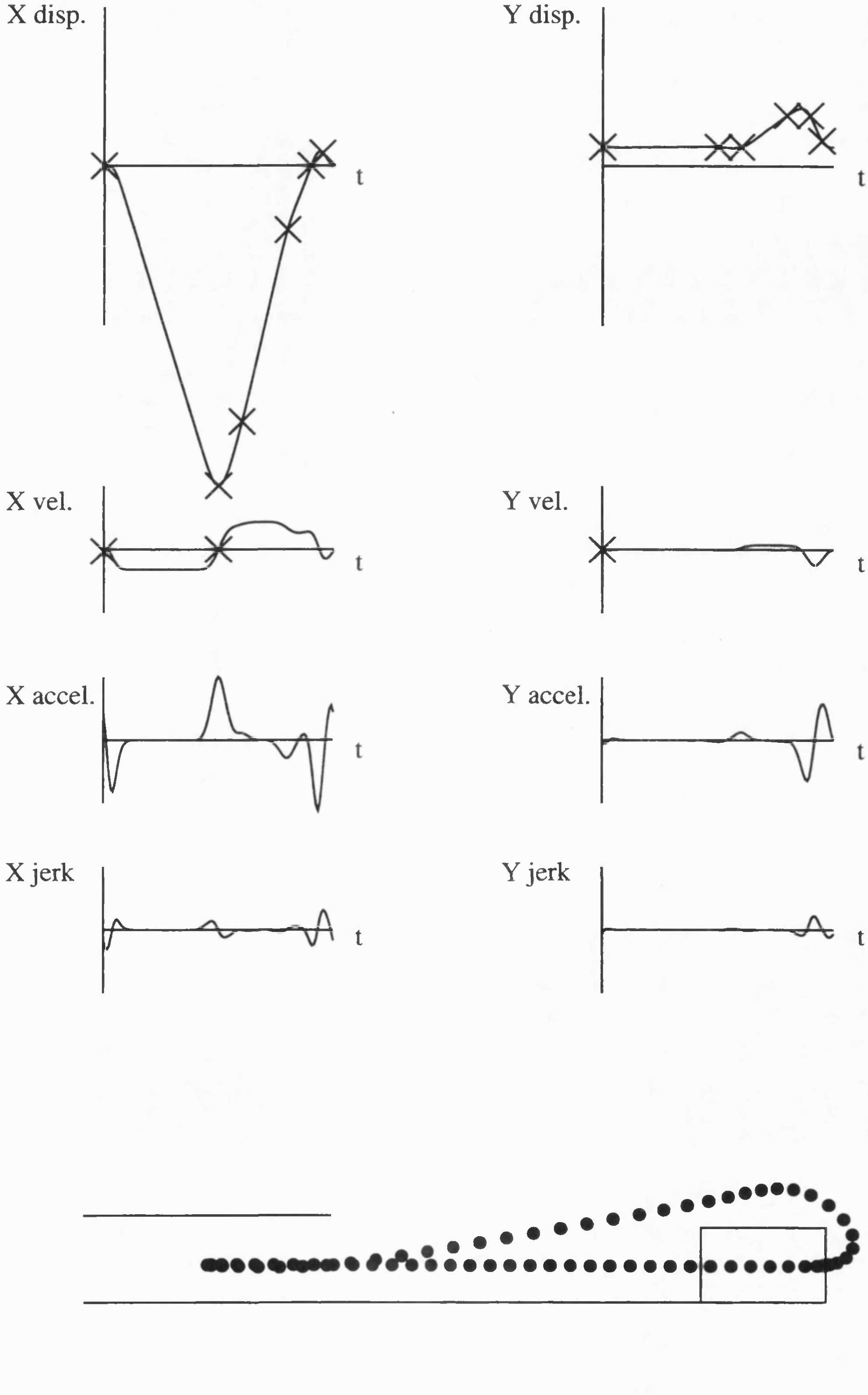


Figure 9.4 Initial guess for arbitrary precision points

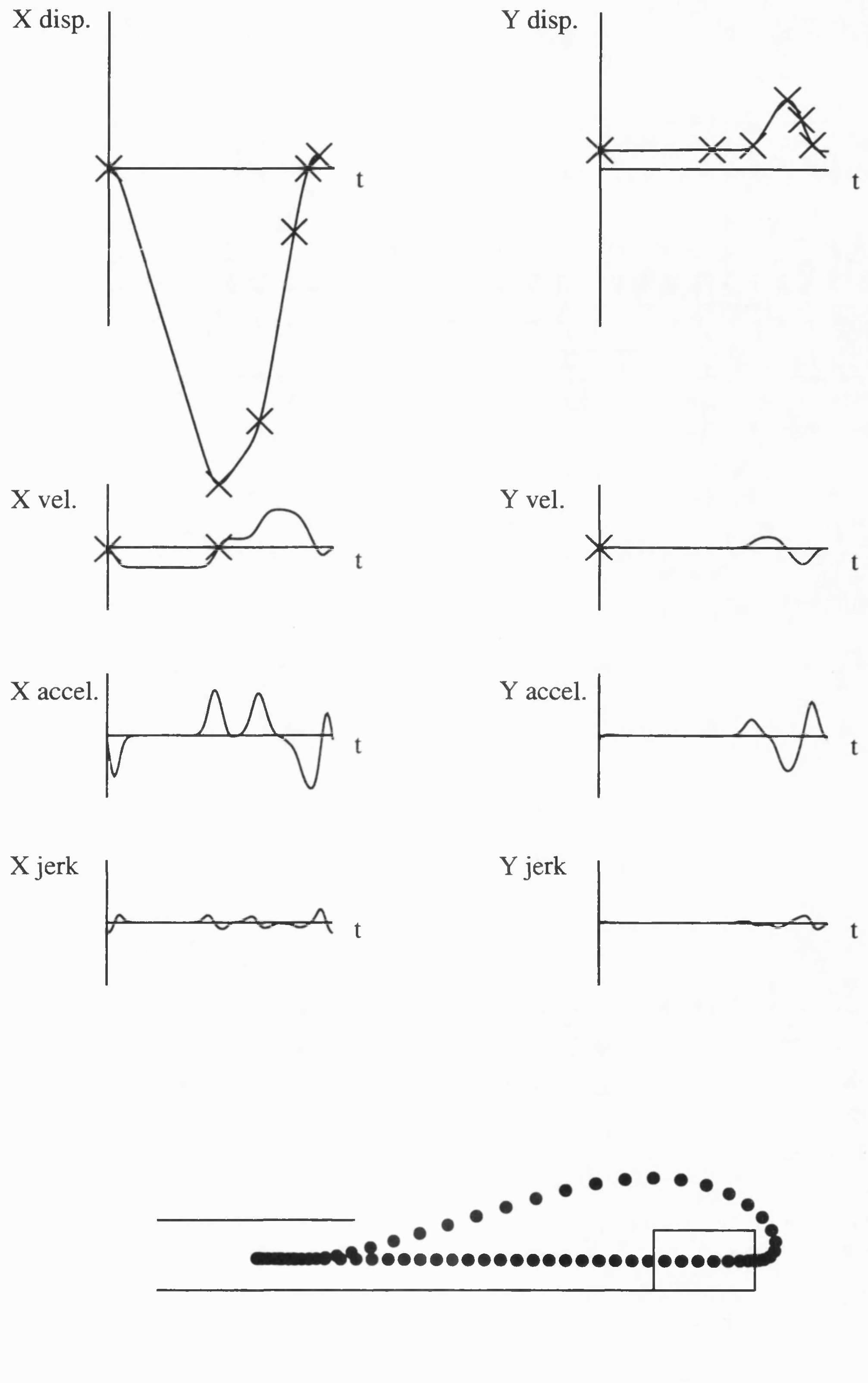


Figure 9.5 Optimised motion

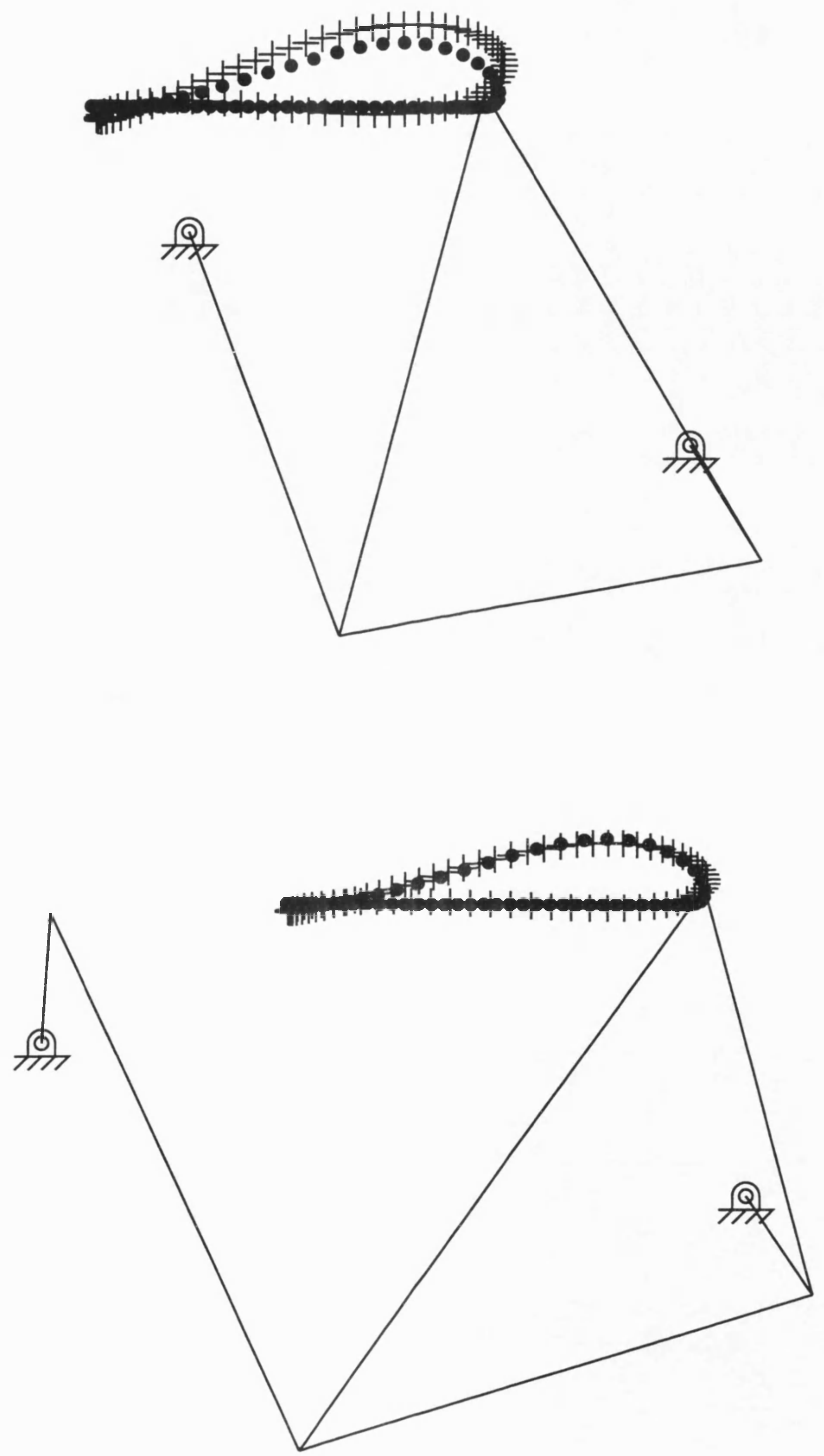


Figure 9.6 Linkages from the constraint modeller's path selection application

Chapter 10 - Conclusions

The motion of packaging machinery is subject to a wide variety of constraints. These have to be taken into account during the conceptual stages of the design process in order to create a machine that can behave correctly. The constraints apply to both the motion of the machine's end-effector and its joints.

In this thesis the motion constraints that are typically imposed on continuous-processing packaging machinery have been examined, and the critical ones have been identified. It has been shown that they can be grouped into the following categories: task-related constraints, clearance constraints, quality of motion, and commercial interests.

A methodology has been proposed to assist in the design of machinery motion. This involves identifying constraints that apply to a particular problem. The approach has been successfully applied in relation to a number of case studies. A crucial part of the methodology is the ability to identify and manipulate important motion constraints. To this end the idea of parametric timing diagrams has been introduced and demonstrated with the case studies.

The variables used within these timing diagrams are manipulated in order to produce instances of the generic motion. Often these are in the form of paths in two-dimensional space, but the techniques are applicable in higher dimensions. The aim of manipulating the variables firstly is to produce valid motions. Secondly it is to allow the motions to be made in some sense optimal.

Having created them, the motions can then be applied to existing mechanical systems for feasibility studies, or as information for use in the synthesis of a new mechanism capable of reproducing them. Depending on the system being designed, there may be a need to use optimisation techniques to improve the combination of the motion and its mechanism.

A number of case studies have been used to illustrate the use of the methodology. These include the design of a system to fill bottles with liquid products, a mechanism to cut extruded ice-cream bars, a material handling device to load blank cans onto a rotating forming machine, and a pushing mechanism to load boxes into labelling machine. It has been shown with these case studies that the methodology is applicable to the design of cam-driven systems, linkage mechanisms, and systems comprising programmable drives. It also fits in with existing path selection techniques.

There are many benefits of using a parametric form of the timing diagrams. They give the ability to generate motions of a similar form easily to account for changes in the product being processed. This was shown in the ice-cream cutter design, the filling machine, and the box pushing mechanism. They are therefore useful for archival purposes and for reuse of knowledge in future design projects.

In some cases it is necessary to make arbitrary selection of variables, or to introduce `artificial' precision point constraints on the timing diagrams purely to make further progress possible. When this happens it is necessary to use optimisation to smooth the motions or minimise the over-constraining effect they have.

The thought processes required to produce the diagrams can help to identify gaps in an engineer's appreciation of the design problem, or to highlight significant features in the required motion. The former is important in order to produce acceptable designs. The latter encourages investigation of the range of performance that can be achieved by a particular design concept under different sets of constraints.

References

Ahrikencheikh, C., Seireg, A.A., Ravani, B. (1994) Optimal and conforming motion of a point in a constrained plane. *Trans. ASME, Journal of Mechanical Design*, Vol. 116, 474-479

Anon. (1996) *MOTION Users' Guide*, Limaçon Ltd.

Bawab, S., Sabada, S., Kinzel, G.L., Waldron, K.J. (1997) Automatic synthesis of crank driven four-bar mechanisms for two, three, or four-position motion generation. *Trans. ASME, Journal of Mechanical Design*, Vol. 119, 225-231

Bobrow, J.E., Dubowsky, S., Gibson, J.S. (1985) Time-optimal control of robotic manipulators along specified paths. *International Journal of Robotics Research*, Vol. 4, No. 3, 3-17

Chen, F.Y. (1982) *Mechanics and design of cam mechanisms*, Pergamon Press

Erdman, A.G. (1995) Computer-aided mechanism design: now and the future. *Trans. ASME, Journal of Mechanical Design*, Vol. 117, 93-100

Ge, Q.J., Ravani, B. (1994) Geometric construction of Bezier motions. *Trans. ASME, Journal of Mechanical Design*, Vol. 116, 749-755

Ge, Q.J., Ravani, B. (1994) Computer-aided geometric design of motion interpolants. *Trans. ASME, Journal of Mechanical Design*, Vol. 116, 756-762

Jüttler, B., Wagner, M.G. (1996) Computer-aided-design with spatial rational B-spline motions. *Trans. ASME, Journal of Mechanical Design*, Vol. 118, 193-201

Kenney, L.P.J. (1993) A study of software tools and systems for machinery design. *DTI Grant No. EC/4087/92, final report*

Kenney, L.P.J., Rentoul, A.H., Twyman B.R., Kerr, D.R., Mullineux, G. (1997) A software environment for conceptual mechanism design. *Proc. Inst. Mech. Engrs. Part C, Journal of Mechanical Engineering Science*, Vol. 211, 617-625

Kim, K-Y., Kwak B-M. (1995) A modified selective precision synthesis technique for planar four-bar rigid-body guidance. *Proc. Inst. Mech. Engrs. Part C, Journal of Mechanical Engineering Science*, Vol. 209, 39-47

Kirecci, A. (1993) *Motion design for high-speed machines*, Ph.D. Thesis, Liverpool John Moores University

Kirecci, A., Gilmartin, M.J. (1994) Improved trajectory planning using arbitrary power polynomials. *Proc. Inst. Mech. Engrs. Part I, Journal of Systems and Control Engineering*, Vol. 208, 3-13

MacCarthy, B.L. (1988) Quintic splines for kinematic design, *Computer-Aided Design*, Vol. 20, No. 7, 406-415

Mashford, K.J. (1992) *A methodology for the concurrent design of cam driven systems with particular application to motion synthesis*. Ph.D. Thesis, Brunel University

McGarva, J.R., Mullineux, G. (1992) A new methodology for rapid synthesis of function generators. *Proc. Inst. Mech. Engrs. Part C, Journal of Mechanical Engineering Science*, Vol. 206, 391-398

McGarva J.R., Mullineux, G. (1994) The implementation of closed B-spline curves for application to mechanisms. *Computers in Industry*, Vol. 27, 287-290

McGarva, J.R. (1994) Rapid search and selection of path generating mechanisms from a library. *Mechanism and Machine Theory*, Vol. 29, No. 2, 223-235

McGarva, J.R. (1995) Non-uniform motion by mechanical means. *Proc. Inst. Mech. Engrs. Part B, Journal of Engineering Manufacture*, Vol. 209, 295-303

Medland, A.J., Mullineux, G., Twyman B.R., Rentoul, A.H. (1995) A constraint-based approach to the design and optimisation of mechanism systems. *Proc. IUTAM Symposium on Optimization of Mechanical Systems, Stuttgart*, 205-212

Medland, A.J., Mullineux, G., Rentoul, A.H., Twyman B.R. (1996) Software for constraint-based design. *Proc. Mechatronics 96 with M2VIP96, Guimaraes*, Vol. 1, 349-354

Medland, A.J., Mullineux, G. (1991) Application of constraint modelling techniques to the design and analysis of high-speed packaging machinery. *Proc. Inst. Mech. Engrs., European Engineering Research and Technology Transfer Congress*, 213-218

Ozaki, H., Lin, C. (1996) Optimal B-spline joint trajectory generation for collision-free movements of a manipulator under dynamic constraints. *Proc. 1996 IEEE International Conference on Robotics and Automation, Minneapolis*, 3592-3597

Rees Jones, J., Rooney, G.T., Fischer, P.J. (1984) Dynamically smoother motion for industrial robots: design and implementation. *Proc. I.Mech.E. Conference - UK Robotics Research*, 59-63

Roston, G.P., Sturges, R.H. (1996) Genetic algorithm synthesis of four-bar mechanisms. *Artificial Intelligence for Engineering Design, Analysis and Manufacturing*, Vol.. 10, 371-390

Sandor, G.N., Erdman, A.G. (1984) *Advanced mechanism design: analysis and synthesis-volume 2*, Prentice Hall

Seaward, D.R., Vernon, G.W. (1994) Design of high-speed process machinery transmissions in order to maximise the benefits of servo drives. *Proc. Inst. Mech. Engrs. Part I, Journal of Systems and Control Engineering*, Vol. 208, 1994, 15-29